## 1 GPUs and SIMD [90 points]

We define the SIMD utilization of a program run on a GPU as the fraction of SIMD lanes that are kept busy with active threads during the run of a program. As we saw in lecture and practice exercises, the SIMD utilization of a program is computed across the complete run of the program.

The following code segment is run on a GPU. Each thread executes a single iteration of the shown loop. Assume that the data values of the arrays A, B, and C are already in vector registers, so there are no loads and stores in this program. (Hint: Notice that there are 6 instructions in each thread.) A warp in the GPU consists of 32 threads, and there are 32 SIMD lanes in the GPU. Please assume that all values in arrays B and C have magnitudes less than 10 (i.e., $|B[i]|<10$ and $|C[i]|<10$, for all i).

```
for (i = 0; i < 1008; i++) {
    A[i] = B[i] * C[i];
    if (A[i] < O) {
        C[i] = A[i] * B[i];
        if (C[i] < 0) {
            A[i] = A[i] + 1;
        }
        A[i] = A[i] - 2;
    }
}
```

Please answer the following six questions.
(a) [10 points] How many warps does it take to execute this program?

32 warps
Explanation: number of warps $=\lceil$ (number of elements) $/($ warp size $)\rceil=\lceil 1008 / 32\rceil$ $=32$ warps
(b) [10 points] What is the maximum possible SIMD utilization of this program?
0.984375

Explanation: We have 31 fully-utilized warps and one warp with only 16 active threads. In case with no branch divergence, the SIMD utilization would be: $(32 \times 31+16) /(32 \times 32)=0.984375$
(c) [20 points] Please describe what needs to be true about arrays B and C to reach the maximum possible SIMD utilization asked in part (b). (Please cover all possible cases in your answer)

For each 32 consecutive elements: $((\mathrm{B}[\mathrm{i}]==0 \| \mathrm{C}[\mathrm{i}]==0)\|(\mathrm{B}[\mathrm{i}]<0 \& \mathrm{C}[\mathrm{i}]<0)\|(\mathrm{B}[\mathrm{i}]>0 \&$ $\mathrm{C}[\mathrm{i}]>0)) \|((\mathrm{B}[\mathrm{i}]<0 \& \mathrm{C}[\mathrm{i}]>0) \|(\mathrm{B}[\mathrm{i}]>0 \& \mathrm{C}[\mathrm{i}]<0))$

Explanation: We can have two possibilities:
(1) No threads inside a warp take the first "if". It is possible if for 32 consecutive elements, the following condition is true:
$(\mathrm{B}[\mathrm{i}]==0 \| \mathrm{C}[\mathrm{i}]==0)\|(\mathrm{B}[\mathrm{i}]<0 \& \mathrm{C}[\mathrm{i}]<0)\|(\mathrm{B}[\mathrm{i}]>0 \& \mathrm{C}[\mathrm{i}]>0)$
(2) All threads inside a warp take the first "if". It is possible if for 32 consecutive elements, the following condition is true:
$(\mathrm{B}[\mathrm{i}]<0 \& \mathrm{C}[\mathrm{i}]>0) \|(\mathrm{B}[\mathrm{i}]>0 \& \mathrm{C}[\mathrm{i}]<0)$
This condition also ensures that for the second "if", all threads take the branch or no threads take the branch.
Finally, for every 32 consecutive elements, we should have one of the mentioned possibilities. For the warp with 16 active threads, we should have one of mentioned possibilities for every 16 consecutive elements
(d) [10 points] What is the minimum possible SIMD utilization of this program?
$((32+32+4) \times 31+16+16+4) /(32 \times 6 \times 32)=0.3489$
Explanation: The first two lines must be executed by every thread in a warp. The minimum utilization results when a single thread from each warp passes both conditions, and every other thread fails to meet the condition on the first "if". The thread per warp that meets both conditions, executes four instructions.
(e) [20 points] Please describe what needs to be true about arrays B and C to reach the minimum possible SIMD utilization asked in part (d). (Please cover all possible cases in your answer)

Exactly 1 of every 32 consecutive elements in arrays B and C should have this condition: $\mathrm{B}[\mathrm{i}]>$ $0 \& \mathrm{C}[\mathrm{i}]<0$

Explanation: Only one thread in a warp should pass the first condition. Therefore, exactly 1 of every 32 consecutive elements in arrays B and C should have the following condition: $(\mathrm{B}[\mathrm{i}]<0 \& \mathrm{C}[\mathrm{i}]>0) \|(\mathrm{B}[\mathrm{i}]>0 \& \mathrm{C}[\mathrm{i}]<0)$
However, the thread in a warp which passes the first condition should also pass the second condition. Therefore, the only condition which ensures the minimum possible SIMD utilization is: exactly 1 of every 32 consecutive elements in arrays B and C should have this condition: $\mathrm{B}[\mathrm{i}]$ $>0 \& \mathrm{C}[\mathrm{i}]<0$. For the warp with 16 active threads, this condition should be true for exactly 1 of the 16 elements.
(f) [20 points] Now consider a GPU that employs Dynamic Warp Formation ( $D W F$ ) to improve the SIMD utilization. As we discussed in the class, DWF dynamically merges threads executing the same instruction (after branch divergence). What is the maximum achievable SIMD utilization using DWF under the conditions you found in part (e)? Explain your answer.
$((32+32) \times 31+16+16+32 \times 4) /((32+32) \times 32+32 \times 4)=0.985$
Explanation: DWF can ideally merge 32 warps with only one active threads in a single warp. This could be happen if none of the threads inside the warp have overlaps. If it happens, we will run 31 fully utilized warps and one warp with 16 active threads for the first and second instructions. Then, we will have only one fully-utilized warp which executes the remaining 4 instructions.

## 2 In-DRAM Bitmap Indices [70 points]

Recall that in class we discussed Ambit, which is a DRAM design that can greatly accelerate Bulk Bitwise Operations by providing the ability to perform bitwise AND/OR of two rows in a subarray.

One real-world application that can benefit from Ambit's in-DRAM bulk bitwise operations is the database bitmap index, as we also discussed in the lecture. By using bitmap indices, we want to run the following query on a database that keeps track of user actions: "How many unique users were active every week for the past $w$ weeks?" Every week, each user is represented by a single bit. If the user was active a given week, the corresponding bit is set to 1 . The total number of users is $u$.

We assume the bits corresponding to one week are all in the same row. If $u$ is greater than the total number of bits in one row (the row size is 8 kilobytes), more rows in different subarrays are used for the same week. We assume that all weeks corresponding to the users in one subarray fit in that subarray.

We would like to compare two possible implementations of the database query:

- CPU-based implementation: This implementation reads the bits of all $u$ users for the $w$ weeks. For each user, it ands the bits corresponding to the past $w$ weeks. Then, it performs a bit-count operation to compute the final result.
Since this operation is very memory-bound, we simplify the estimation of the execution time as the time needed to read all bits for the $u$ users in the last $w$ weeks. The memory bandwidth that the CPU can exploit is $X$ bytes/s.
- Ambit-based implementation: This implementation takes advantage of bulk and operations of Ambit. In each subarray, we reserve one Accumulation row and one Operand row (besides the control rows that are needed for the regular operation of Ambit). Initially, all bits in the Accumulation row are set to 1. Any row can be moved to the Operand row by using RowClone (recall that RowClone is a mechanism that enables very fast copying of a row to another row in the same subarray). $t_{r c}$ and $t_{\text {and }}$ are the latencies (in seconds) of RowClone's copy and Ambit's and respectively.
Since Ambit does not support bit-count operations inside DRAM, the final bit-count is still executed on the CPU. We consider that the execution time of the bit-count operation is negligible compared to the time needed to read all bits from the Accumulation rows by the CPU.
(a) [15 points] What is the total number of DRAM rows that are occupied by $u$ users and $w$ weeks?

$$
\text { TotalRows }=\left\lceil\frac{u}{8 \times 8 k}\right\rceil \times w .
$$

## Explanation:

The $u$ users are spread across a number of subarrays:
NumSubarrays $=\left\lceil\frac{u}{8 \times 8 k}\right\rceil$.
Thus, the total number of rows is:
TotalRows $=\left\lceil\frac{u}{8 \times 8 k}\right\rceil \times w$.
(b) [20 points] What is the throughput in users/second of the Ambit-based implementation?
$T h r_{A m b i t}=\frac{u}{\Gamma \frac{u}{8 \times 8 k} \left\lvert\, \times w \times\left(t_{r c}+t_{a n d}\right)+\frac{u}{X \times 8}\right.}$ users/second.

## Explanation:

First, let us calculate the total time for all bulk and operations. We should add $t_{r c}$ and $t_{a n d}$ for all rows:
$t_{\text {and-total }}=\left\lceil\frac{u}{8 \times 8 k}\right\rceil \times w \times\left(t_{r c}+t_{\text {and }}\right)$ seconds.
Then, we calculate the time needed to compute the bit count on CPU:
$t_{\text {bitcount }}=\frac{\frac{u}{s}}{X}=\frac{u}{X \times 8}$ seconds.
Thus, the throughput in users/s is:
$T h r_{\text {Ambit }}=\frac{u}{t_{\text {and-total }}+t_{\text {bitcount }}}$ users/second.
(c) [20 points] What is the throughput in users/second of the CPU implementation?
$T h r_{C P U}=\frac{X \times 8}{w}$ users $/$ second.

## Explanation:

We calculate the time needed to bring all users and weeks to the CPU:
$t_{C P U}=\frac{\frac{u \times w}{8}}{X}=\frac{u \times w}{X \times 8}$ seconds.
Thus, the throughput in users/s is:
$T h r_{C P U}=\frac{u}{t_{C P U}}=\frac{X \times 8}{w}$ users $/$ second.
(d) [15 points] What is the maximum $w$ for the CPU implementation to be faster than the Ambit-based implementation? Assume $u$ is a multiple of the row size.
$w<\frac{1}{1-\frac{X}{8 k} \times\left(t_{r c}+t_{\text {and }}\right)}$.

## Explanation:

We compare $t_{C P U}$ with $t_{\text {and-total }}+t_{\text {bitcount }}$ :
$t_{C P U}<t_{\text {and-total }}+t_{\text {bitcount }}$;
$\frac{u \times w}{X \times 8}<\frac{u}{8 \times 8 k} \times w \times\left(t_{r c}+t_{a n d}\right)+\frac{u}{X \times 8} ;$
$w<\frac{1}{1-\frac{X}{8 k} \times\left(t_{r c}+t_{\text {and }}\right)}$.

## 3 Cache Performance Analysis [80 points]

We are going to microbenchmark the cache hierarchy of a computer with the following two codes. The array data contains 32-bit unsigned integer values. For simplicity, we consider that accesses to the array latency bypass all caches (i.e., latency is not cached). timer() returns a timestamp in cycles.

```
(1) j = 0;
    for (i=0; i<size; i+=stride){
            start = timer();
            d = data[i];
            stop = timer();
            latency[j++] = stop - start;
    }
(2) for (i=0; i<size1; i+=stride1){
        d = data[i];
    }
    j = 0;
    for (i=0; i<size2; i+=stride2) {
        start = timer();
        d = data[i];
        stop = timer();
        latency[j++] = stop - start;
    }
```

The cache hierarchy has two levels. L1 is a 4 kB set associative cache.
(a) [15 points] When we run code (1), we obtain the latency values in the following chart for the first 64 reads to the array data (in the first 64 iterations of the loop) with stride equal to 1 . What are the cache block sizes in L1 and L2?


L1 cache block size is 32 bytes, and L2 cache block size is 128 bytes.

## Explanation:

The highest latency values ( 700 cycles) correspond to L2 cache misses. The latency of 300 cycles correspond to L1 cache misses that hit the L2 cache. The lowest latency ( 100 cycles) corresponds to L1 cache hits. We find L1 misses every 8 32-bit elements, and L2 misses every 32 32-bit elements. Thus, L1 cache blocks are of size 32 bytes, while L2 cache blocks are 128 bytes.
(b) [20 points] Using code (2) with stride1 $=$ stride $2=32$, size1 $=1056$, and size $2=1024$, we observe latency [0] $=300$ cycles. However, if size1 $=1024$, latency [ 0 ] $=100$ cycles. What is the maximum number of ways in L1? (Note: The replacement policy can be either FIFO or LRU).

The maximum number of ways is 32 .

## Explanation:

In L1 there are a total of 128 cache blocks. If the accessed space is 1056 elements, 33 cache blocks are read. In case of having more than 32 ways, there would be a hit in the second loop. However, with 32 or less ways, the cache block that contains data [0] is replaced in the last iteration of the first loop.
(c) [20 points] We want to find out the exact replacement policy, assuming that the associativity is the maximum obtained in part (b). We first run code (2) with stride1 $=32$, size1 $=1024$, stride $2=64$, and size $2=1056$. Then (after resetting $j$ ), we run code ( 1 ) with stride $=32$ and size $=1024$. We observe latency[1] = 100 cycles. What is the replacement policy? Explain. (Hint: The replacement policy can be either FIFO or LRU. You need to find the correct one and explain).

## It is FIFO.

## Explanation:

In the second loop of code (2), the last cache block that is accessed (data [1024]) replaces the cache block that contains data [0], if the replacement policy is FIFO. If the replacement policy is LRU, the LRU cache block (the one that contains data [32]). Because we observe that latency [1] is 100 cycles, and it corresponds to the access to data [32], we conclude the replacement policy is FIFO.
(d) [25 points] Now we carry out two consecutive runs of code (1) for different values of size. In the first run, stride is equal to 1 . In the second run, stride is equal to 16 . We ignore the latency results of the first run, and average the latency results of the second run. We obtain the following graph. What do the four parts shown with the arrows represent?


Before arrow 1:

The entire array fits in L1. In arrow 1 the size of the array is the same as the size of L1 ( 4 kB ).

Between arrow 1 and arrow 2:

Some accesses in the second run hit in L1 and other accesses hit in L2.

Between arrow 2 and arrow 3:

All accesses in the second run hit in L2.

Between arrow 3 and arrow 4:

Still some accesses in the second run hit in L2. Arrow 3 marks the size of the L2 cache.

After arrow 4:

The accesses of the second run always miss in L2, and it is necessary to access main memory.

Explain as needed (if you need more):

## 4 SIMD [90 points]

We have two SIMD engines: 1) a traditional vector processor and 2) a traditional array processor. Both processors can support a vector length up to 16 .

All instructions can be fully pipelined, the processor can issue one vector instruction per cycle, and the pipeline does not forward data (no chaining). For the sake of simplicity, we ignore the latency of the pipeline stages other than the execution stages (e.g, decode stage latency: 0 cycles, write back latency: 0 cycles, etc).

We implement the following instructions in both designs, with their corresponding execution latencies:

| Operation | Description | Name | Latency of a single operation (VLEN=1) |
| :--- | :--- | :--- | :--- |
| VADD | VDST $\leftarrow$ VSRC1 + VSRC2 | vector add | 5 cycles |
| VMUL | VDST $\leftarrow$ VSRC1 $*$ VSRC2 | vector mult. | 15 cycles |
| VSHR | VDST $\leftarrow$ VSRC $\gg 1$ | vector shift | 1 cycles |
| VLD | VDST $\leftarrow$ mem[SRC] | vector load | 20 cycles |
| VST | VSRC $\rightarrow$ mem[DST] | vector store | 20 cycles |

- All the vector instructions operate with a vector length specified by VLEN. The VLD instruction loads VLEN consecutive elements from the DST address specified by the value in the VDST register. The VST instruction stores VLEN elements from the VSRC register in consecutive addresses in memory, starting from the address specified in DST.
- Both processors have eight vector registers (VR0 to VR7) which can contain up to 16 elements, and eight scalar registers (R0 to R7). The entire vector register needs to be ready (i.e., populated with all VLEN elements) before any element of it can be used as part of another operation.
- The memory can sustain a throughput of one element per cycle. The memory consists of 16 banks that can be accessed independently. A single memory access can be initiated in each cycle. The memory can sustain 16 parallel accesses if they all go to different banks.
(a) [10 points] Which processor (array or vector processor) is more costly in terms of chip area? Explain.

Array processor
Explanation: An array processor requires 16 functional units for an operation whereas a vector processor requires only 1.
(b) [25 points] The following code takes 52 cycles to execute on the vector processor:

VADD VR2 $\leftarrow$ VR1, VR0
VADD VR3 $\leftarrow$ VR2, VR5
VMUL VR6 $\leftarrow$ VR2, VR3
What is the VLEN of the instructions? Explain your answer.
VLEN: 10
Explanation: $5+($ VLEN-1 $)+5+($ VLEN- 1$)+15+($ VLEN -1$)=52 \Rightarrow$ VLEN $=10$
How long would the same code execute on an array processor with the same vector length?

## 25 cycles

Explanation: there are data dependencies among instructions $\Rightarrow 5+5+15=25$ cycles
(c) [25 points] The following code takes 94 cycles to execute on the vector processor:

```
VLD \(\quad \mathrm{VR} 0 \leftarrow \operatorname{mem}[\mathrm{R} 0]\)
VLD VR1 \(\leftarrow \operatorname{mem}[\mathrm{R} 1]\)
VADD VR2 \(\leftarrow\) VR1, VR0
VSHR VR2 \(\leftarrow \mathrm{VR} 2\)
VST VR2 \(\rightarrow\) mem[R2]
```

Assume that the elements loaded in VR0 are all placed in different banks, and that the elements loaded into VR1 are placed in the same banks as the elements in VR0. Similarly, the elements of VR2 are stored in different banks in memory. What is the VLEN of the instructions? Explain your answer.

VLEN: 8
Explanation: $20+20+($ VLEN-1 $)+5+($ VLEN -1$)+1+($ VLEN -1$)+20+($ VLEN -1$)=94$.
$\Rightarrow$ VLEN $=8$
(d) [30 points] We replace the memory with a new module whose characteristics are unknown. The following code (the same as that in (c)) takes 163 cycles to execute on the vector processor:

```
\(\mathrm{VLD} \quad \mathrm{VR} 0 \leftarrow \operatorname{mem}[\mathrm{R} 0]\)
VLD VR1 \(\leftarrow \operatorname{mem}[\mathrm{R} 1]\)
VADD VR2 \(\leftarrow\) VR1, VR0
VSHR VR2 \(\leftarrow \mathrm{VR} 2\)
VST VR2 \(\rightarrow\) mem[R2]
```

The VLEN of the instructions is 16 . The elements loaded in VR0 are placed in consecutive banks, the elements loaded in VR1 are placed in consecutive banks, and the elements of VR2 are also stored in consecutive banks. What is the number of banks of the new memory module? Explain.
[Correction] The number of cycles should be 170 instead of 163 . For grading this question the instructor took into account only the student's reasoning.

Number of banks: 8
Explanation: Assuming that the number of banks is power of two, $20^{*}(16 /$ banks $)+$ $20^{*}(16 /$ banks $)+($ banks- 1$)+5+($ VLEN-1 $)+1+($ VLEN-1 $)+20^{*}(16 /$ banks $)+($ banks- 1$)=170$ $\Rightarrow$ banks $=8$

## 5 DRAM Refresh [80 points]

A memory system is composed of eight banks, and each bank contains 32 K rows. The row size is 8 KB .
Every DRAM row refresh is initiated by a command from the memory controller, and it refreshes a single row. Each refresh command keeps the command bus busy for 5 ns .

We define command bus utilization as a fraction of the total time during which the command bus is busy due to refresh.

The retention time of each row depends on the temperature $(\mathrm{T})$. The rows have different retention times, as shown in the following Table 1:

| Retention Time | Number of rows |
| :---: | :--- |
| $(128-T) \mathrm{ms}, 0^{\circ} \mathrm{C} \leq T \leq 128^{\circ} \mathrm{C}$ | $2^{8}$ rows |
| $2 *(128-T) \mathrm{ms}, 0^{\circ} \mathrm{C} \leq T \leq 128^{\circ} \mathrm{C}$ | $2^{16}$ rows |
| $4 *(128-T) \mathrm{ms}, 0^{\circ} \mathrm{C} \leq T \leq 128^{\circ} \mathrm{C}$ | all other rows |
| $8 *(128-T) \mathrm{ms}, 0^{\circ} \mathrm{C} \leq T \leq 128^{\circ} \mathrm{C}$ | $2^{8}$ rows |

Table 1: Retention time

### 5.1 Refresh Policy A [20 points]

Assume that the memory controller implements a refresh policy where all rows are refreshed with a fixed refresh interval, which covers the worst-case retention time (Table 1).
(a) [5 points] What is the maximum temperature at which the DRAM can operate reliably with a refresh interval of 32 ms ?

$$
\mathrm{T}=96^{\circ} \mathrm{C} .
$$

Explanation. Looking at the first column of Table 1, it is obvious that the DRAM rows with the least retention time (i.e., the first row of the table) can operate at temperatures up to $96^{\circ} \mathrm{C}$ (128-96) when refreshed at fixed 32 ms refresh period.
(b) [15 points] What command bus utilization is directly caused by DRAM refreshes (with refresh interval of $32 \mathrm{~ms})$ ?
$4.096 \%$.
Explanation. The time that the command bus spends on refresh commands in 32 ms is $2^{18 *} 5 \mathrm{~ns}$, where $2^{18}$ is the number of rows ( 8 banks, 32 K rows per banks), and 5 ns is the time that the bus is busy for each refresh command.
Utilization $=\frac{2^{18} * 5 n s}{32 m s}=4.096 \%$

### 5.2 Refresh Policy B [15 points]

Now assume that the memory controller implements a refresh policy where all rows are refreshed only as frequently as required to correctly maintain their data (Table 1).
(a) [15 points] How many refreshes are performed by the memory controller during a 1.024 second period? (with $\mathrm{T}=64^{\circ} \mathrm{C}$ )

The number of refreshes in 1.024 seconds is 1313280 .
Explanation. $2^{8}$ rows are refreshed every $64 \mathrm{~ms} \Longrightarrow(1024 / 64) * 2^{8}=2^{12}$ refresh commands in 1.024 seconds.
$2^{16}$ rows are refreshed every $128 \mathrm{~ms} \Longrightarrow(1024 / 128) * 2^{16}=2^{19}$ refresh commands in 1.024 seconds.
$2^{8}$ rows are refreshed every $512 \mathrm{~ms} \Longrightarrow(1024 / 512) * 2^{8}=2^{9}$ refresh commands in 1.024 seconds.
The remaining rows are $2^{18}-\left(2^{8}+2^{16}+2^{8}\right)=2^{18}-2^{16}-2^{9}$, so: $2^{18}-2^{16}-2^{9}$ rows are refreshed every $256 \mathrm{~ms} \Longrightarrow(1024 / 256) *\left(2^{18}-2^{16}-2^{9}\right)=2^{20}-2^{18}-2^{11}$ refresh commands in 1.024 second.

Total: $2^{12}+2^{19}+2^{9}+2^{20}-2^{18}-2^{11}=1313280$ refresh commands in 1.024 second.

### 5.3 Refresh Policy C [25 points]

Assume that the memory controller implements an even smarter policy to refresh the memory. In this policy, the refresh interval is fixed, and it covers the worst-case retention time ( 64 ms ), as the refresh policy in part 5.1. However, as an optimization, a row is refreshed only if it has not been accessed during the past refresh interval. For maintaining correctness, if a cell reaches its maximum retention time without being refreshed, the memory controller issues a refresh command.
(a) [5 points] Why does a row not need to be refreshed if it was accessed in the past refresh interval?

The charge of a DRAM cell is restored when it is accessed.
(b) [20 points] A program accesses all the rows repeatedly in the DRAM. The following table shows the access interval of the rows, the number of rows accessed with the corresponding access interval, and the retention times of the rows that are accessed with the corresponding interval.

| Access interval | Number of rows | Retention times |
| :--- | :--- | :--- |
| 1 ms | $2^{16}$ rows | $64 \mathrm{~ms}, 128 \mathrm{~ms}$ or 256 ms |
| 60 ms | $2^{16}$ rows | $64 \mathrm{~ms}, 128 \mathrm{~ms}$ or 256 ms |
| 128 ms | all other rows | 128 ms or 256 ms |

What command bus utilization is directly caused by DRAM refreshes?

Command bus utilization: $0.512 \%$.
Explanation. The rows that are accessed every 1 ms and every 60 ms do not need to be refreshed.

The remaining rows $\left(2^{17}\right)$ are accessed once every two refresh intervals ( 128 ms ). So, the command bus utilization is $\frac{2^{17} * 5 n s * 100}{128 \mathrm{~ms}}=0.512 \%$

### 5.4 Refresh Policy D [20 points]

Assume that the memory controller implements an approximate mechanism to reduce refresh rate using Bloom filters, as we discussed in class. For this question we assume the retention times in Table 1 with a constant temperature of $64^{\circ} \mathrm{C}$.

One Bloom filter is used to represent the set of all rows that require a 64 ms refresh rate.
The memory controller's refresh logic is modified so that on every potential refresh of a row (every 64 ms ), the refresh logic probes the Bloom filter. If the Bloom filter probe results in a "hit" for the row address, then the row is refreshed. Any row that does not hit in the Bloom filter is refreshed at the default rate of once per 128 ms .
(a) [20 points] The memory controller performs 2107384 refreshes in total across the channel over a time interval of 1.024 seconds. What is the false positive rate of the Bloom filter? (NOTE: False positive rate $=$ Total number of false positives / Total number of accesses).

- Hint: $2107384=2^{3} *\left(2^{18}+2^{11}-2^{10}+2^{9}-2^{8}-1\right)$

False Positive rate $=\left(2^{10}-1\right) / 2^{18}$.
Explanation. First, there are $2^{8}$ rows that need to be refreshed every 64 ms . All the other rows $\left(2^{18}-2^{8}\right)$ will be refreshed every 128 ms .

At 64 ms , we have $2^{8}$ rows which actually require this rate, plus P out of the $2^{18}-2^{8}$ other rows which will be false-positives. In a period of 1.024 s , this bin is refreshed 16 times. The number of refreshes for this bin is $16 *\left(2^{8}+P *\left(2^{18}-2^{8}\right)\right)$.

At 128 ms we have the rest of the rows which are $2^{18}-2^{8}$ minus the false positives refreshed in the 64 ms bin. So, the number of rows are $2^{18}-2^{8}-P *\left(2^{18}-2^{8}\right)$. In a period of 1.024 s , this bin is refreshed 8 times. The number of refreshes for this bin is $8 *\left(\left(2^{18}-2^{8}\right)-\left(P *\left(2^{18}-2^{8}\right)\right)\right.$

Thus, in total we have $16 *\left(2^{8}+P *\left(2^{18}-2^{8}\right)\right)+8 *\left(\left(2^{18}-2^{8}\right)-P *\left(2^{18}-2^{8}\right)\right)=2107384$. Solving the equation, $\mathrm{P}=2^{-8}$.

Finally, False positive rate $=$ Total number of false positives $/$ Total number of accesses $=$ $\left.P *\left(2^{18}-2^{8}\right)\right) / 2^{18}=2^{-8} *\left(2^{18}-2^{8}\right) / 2^{18}=\left(2^{10}-1\right) / 2^{18}$

## 6 Caching vs. Processing-in-Memory [80 points]

We are given the following piece of code that makes accesses to integer arrays A and B . The size of each element in both A and B is 4 bytes. The base address of array A is $0 x 00001000$, and the base address of B is $0 x 00008000$.

```
movi R1, #0x1000 // Store the base address of A in R1
movi R2, #0x8000 // Store the base address of B in R2
movi R3, #0
Outer_Loop:
    movi R4, #0
    movi R7, #0
    Inner_Loop:
        add R5, R3, R4 // R5 = R3 + R4
        // load 4 bytes from memory address R1+R5
        ld R5, [R1, R5] // R5 = Memory[R1 + R5],
        ld R6, [R2, R4] // R6 = Memory[R2 + R4]
        mul R5, R5, R6 // R5 = R5 * R6
        add R7, R7, R5 // R7 += R5
        inc R4 // R4++
        bne R4, #2, Inner_Loop // If R4 != 2, jump to Inner_Loop
    //store the data of R7 in memory address R1+R3
    st [R1, R3], R7 // Memory[R1 + R3] = R7,
    inc R3 // R3++
    bne R3, #16, Outer_Loop // If R3 != 16, jump to Outer_Loop
```

You are running the above code on a single-core processor. For now, assume that the processor does not have caches. Therefore, all load/store instructions access the main memory, which has a fixed 50 -cycle latency, for both read and write operations. Assume that all load/store operations are serialized, i.e., the latency of multiple memory requests cannot be overlapped. Also assume that the execution time of a non-memory-access instruction is zero (i.e., we ignore its execution time).
(a) [15 points] What is the execution time of the above piece of code in cycles?

```
4000 cycles.
    Explanation: There are 5 memory accesses for each outer loop iteration. The outer loop iterates
    16 times, and each memory access takes }50\mathrm{ cycles.
    16*5*50=4000 cycles
```

(b) [25 points] Assume that a 128-byte private cache is added to the processor core in the next-generation processor. The cache block size is 8 -byte. The cache is direct-mapped. On a hit, the cache services both read and write requests in 5 cycles. On a miss, the main memory is accessed and the access fills an 8 -byte cache line in 50 cycles. Assuming that the cache is initially empty, what is the new execution time on this processor with the described cache? Show your work.

```
900 cycles.
```


## Explanation.

At the beginning A and B conflict in the first two cache lines. Then the elements of A and B go to different cache lines. The total execution time is 1910 cycles.
Here is the access pattern for the first outer loop iteration:
$0-A[0], B[0], A[1], B[1], A[0]$
The first 4 references are loads, the last $(\mathrm{A}[0])$ is a store. The cache is initially empty. We have a cache miss for $\mathrm{A}[0]$. $\mathrm{A}[0]$ and $\mathrm{A}[1]$ is fetched to 0 th index in the cache. Then, $\mathrm{B}[0]$ is a miss, and it is conflicting with $\mathrm{A}[0]$. So, $\mathrm{A}[0]$ and $\mathrm{A}[1]$ are evicted. Similarly, all cache blocks in the first iteration are conflicting with each other. Since we have only cache misses, the latency for those 5 references is $5 * 50=250$ cycles
The status of the cache after making those seven references is:

| Cache Index | Cache Block |
| :---: | :---: |
| 0 | $\Lambda(0,1), \mathrm{B}(0,1), \Lambda(0,1), \mathrm{B}(0,1), \mathrm{A}(0,1)$ |

Second iteration on the outer loop:
$1-A[1], B[0], A[2], B[1], A[1]$

Cache hits/misses in the order of the references:
$H, M, M, H, M$
Latency $=2 * 5+3 * 50=165$ cycles
Cache Status:

- $\mathrm{A}(0,1)$ is in set 0
- $\mathrm{A}(2,3)$ is in set 1
- the rest of the cache is empty
$2-A[2], B[0], A[3], B[1], A[2]$

Cache hits/misses:
$H, M, H, H, H$
Latency : $4 * 5+1 * 50=70$ cycles

Cache Status:

- $\mathrm{B}(0,1)$ is in set 0
- $\mathrm{A}(2,3)$ is in set 1
- the rest of the cache is empty
$3-A[3], B[0], A[4], B[1], A[3]$

Cache hits/misses:
$H, H, M, H, H$
Latency : $4 * 5+1 * 50=70$ cycles

Cache Status:

- $\mathrm{B}(0,1)$ is in set 0
- $\mathrm{A}(2,3)$ is in set 1
- $\mathrm{A}(4,5)$ is in set 2
- the rest of the cache is empty
$4-A[4], B[0], A[5], B[1], A[4]$
Cache hits/misses:
$H, H, H, H, H$
Latency : $5 * 5=25$ cycles

Cache Status:

- $\mathrm{B}(0,1)$ is in set 0
- $\mathrm{B}(2,3)$ is in set 1
- $\mathrm{A}(4,5)$ is in set 2
- the rest of the cache is empty

After this point, single-miss and zero-miss (all hits) iterations are interleaved until the 16th iteration.
Overall Latency:
$165+70+(70+25) * 7=900$ cycles
(c) [15 points] You are not satisfied with the performance after implementing the described cache. To do better, you consider utilizing a processing unit that is available close to the main memory. This processing unit can directly interface to the main memory with a $10-c y c l e ~ l a t e n c y, ~ f o r ~ b o t h ~ r e a d ~ a n d ~ w r i t e ~ o p e r a t i o n s . ~ H o w ~$ many cycles does it take to execute the same program using the in-memory processing units? (Assume that the in-memory processing unit does not have a cache, and the memory accesses are serialized like in the processor core. The latency of the non-memory-access operations is ignored.)

800 cycles.
Explanation: Same as for the processor core without a cache, but the memory access latency is 10 cycles instead of $50.16 * 5 * 10=800$
(d) [15 points] You friend now suggests that, by changing the cache capacity of the single-core processor (in part (b)), she could provide as good performance as the system that utilizes the memory processing unit (in part (c)).
Is she correct? What is the minimum capacity required for the cache of the single-core processor to match the performance of the program running on the memory processing unit?

No, she is not correct.
Explanation: Increasing the cache capacity does not help because doing so cannot eliminate the conflicts to Set 0 in the first two iterations of the outer loop.
(e) [10 points] What other changes could be made to the cache design to improve the performance of the single-core processor on this program?

Increasing the associativity of the cache.
Explanation: Although there is enough cache capacity to exploit the locality of the accesses, the fact that in the first two iterations the accessed data map to the same set causes conflicts. To improve the hit rate and the performance, we can change the address-to-set mapping policy. For example, we can change the cache design to be set-associative or fully-associative.

## 7 Cache Reverse Engineering [60 points]

Consider a processor using a 4-block LRU-based L1 data cache. Starting with an empty cache, an application accesses three L1 cache blocks in the following order, where consecutive numbers (e.g., $n, n+1, n+2, \ldots$ ) represent the starting addresses of consecutive cache blocks in memory:

$$
n \rightarrow n+2 \rightarrow n+4
$$

### 7.1 Part I: Vulnerability [35 points]

A malicious programmer realizes she can reverse engineer the number of sets and ways in the L1 data cache by issuing just two more accesses and observing only the cache hit rate across these two accesses. Assume that she can insert the malicious accesses only after the above three accesses of the program.

1. [15 points] What are the next two cache block she should access? (e.g., $[\mathrm{n}+$ ?, $\mathrm{n}+$ ?])

There are two possible answers:

- $[\mathrm{n} \rightarrow \mathrm{n}+4]$
- $[\mathrm{n} \rightarrow \mathrm{n}+2]$

Explanation. There are three possible set/way configurations, shown below labeled by their respective sets/ways. Each configuration shows a drawing of the cache state after the three initial accesses. Rows and columns represent sets and ways, respectively, and the LRU address is shown for each occupied set:
(a) (4 sets, 1 way)

| $(n+4)_{L R U}$ |
| :---: |
| - |
| $(n+2)_{L R U}$ |
| - |

(b) ( 2 sets, 2 ways)

| $n+4$ | $(n+2)_{L R U}$ |
| :---: | :---: |
| - | - |

(c) ( 1 set, 4 ways)

| $n_{L R U}$ | $n+2$ | $n+4$ | - |
| :--- | :--- | :--- | :--- |

At this point, all three caches have a $100 \%$ miss rate since they started cold. In order to differentiate the three cases with just two more accesses, we need to induce different hit/miss counts in each of the three types of caches. The only way this is possible is if one cache type experiences two hits, another two misses, and the last one hit and one miss.

Only two solutions exist to produce this case:

- $[\mathrm{n} \rightarrow \mathrm{n}+4]$
(a) $n$ miss, $n+4$ miss $=100 \%$ miss rate
(b) $n$ miss, $n+4$ hit $=80 \%$ miss rate
(c) $n$ hit, $n+4$ hit $=60 \%$ miss rate
- $[\mathbf{n} \rightarrow \mathbf{n}+\mathbf{2}]$
(a) $n$ miss, $n+2$ hit $=80 \%$ miss rate
(b) $n$ miss, $n+2$ miss $=100 \%$ miss rate
(c) $n$ hit, $n+2$ hit $=60 \%$ miss rate

2. [10 points] How many L1 sets/ways would there be if the cache hit rate over the 2 extra instructions was:

There are two possible solutions due to the two possible solutions to part (1). They directly encode the hit/miss rates for the last two accesses shown in the solution above:

- Solution 1:

| L1 hit rate | \# sets | \# ways |
| :---: | :---: | :---: |
| $100 \%$ | 1 | 4 |
| $50 \%$ | 2 | 2 |
| $0 \%$ | 4 | 1 |

- Solution 2:

| L1 hit rate | \# sets | \# ways |
| :---: | :---: | :---: |
| $100 \%$ | 1 | 4 |
| $50 \%$ | 4 | 1 |
| $0 \%$ | 2 | 2 |

3. [10 points] What should the next two accesses be if the replacement policy had been Most Recently Used (MRU)? (e.g., $[\mathrm{n}+$ ?, $\mathrm{n}+$ ? $]$ )

There is no solution for just two more accesses because with an MRU policy, no permutation of two more accesses is able to assign a unique L1 hit rate to each of the three types of caches.

### 7.2 Part II: Exploitation [25 points]

Assuming the original cache design (i.e., with an LRU replacement policy) is using a 1 -set (4-way) configuration, the malicious programmer decides to disrupt a high-security banking application, allowing her to transfer large amounts of foreign assets into her own secure Swiss account. By using a carefully designed concurrently running process to interfere with the banking application, she would like to induce slowdown, thereby exposing opportunity for her mischief.

Assume that the unmodified banking application issues the following access pattern, where each number represents $X$ in $n+X$ :
$0 \rightarrow 6 \rightarrow 1 \rightarrow 7 \rightarrow 6 \rightarrow 1 \rightarrow 4 \rightarrow 0 \rightarrow 5 \rightarrow 0 \rightarrow 7 \rightarrow 4 \rightarrow 2 \rightarrow 7 \rightarrow 2 \rightarrow 4$

1. [10 points] What is the unmodified banking application's cache hit rate?
$\square$
2. [15 points] Now, assume that the malicious programmer knows the access pattern of the banking application. Using this information, she is able to inject a single extra cache access in between each of the banking application's accesses (i.e., she can interleave a malicious access pattern with the normal application's execution).
What is the minimum cache hit rate (not counting extra malicious accesses) that the malicious programmer can induce for the banking application?

## $\frac{2}{16}$

Explanation. Since the malicious programmer wants to cause every cache access by the banking application to miss, she must ensure that any block brought into the cache by an application access is evicted before it is accessed again. The ( 1 set, 4 way) configuration causes a set conflict between any two possible blocks, so in order to force an eviction of a given line, she must ensure that four cache accesses to different, unique cache lines are performed. This guarantees that accessing the given line again results in a miss.

Because she can only insert one extra cache access in between each pair of normal application accesses, she cannot force an eviction of the first access in the two sequences "... $0 \rightarrow 5 \rightarrow 0$..." and "... $\rightarrow 7 \rightarrow 2 \ldots$ ". In these two sequences, accesses to the same cache block are separated by only one access. This means that the malicious programmer can only cause a total of three distinct cache block accesses in between the accesses to the same cache block, and therefore, she cannot prevent a cache hit.

Thus, despite the malicious programmer's best effort, the two cache accesses shown below in bold will still hit in the cache, resulting in an application cache hit rate of $\frac{2}{16}$.

$$
0 \rightarrow 6 \rightarrow 1 \rightarrow 7 \rightarrow 6 \rightarrow 1 \rightarrow 4 \rightarrow 0 \rightarrow 5 \rightarrow \mathbf{0} \rightarrow 7 \rightarrow 4 \rightarrow 2 \rightarrow 7 \rightarrow \mathbf{2} \rightarrow 4
$$

Here is an example cache access sequence resulting in the solution hit rate, with the malicious injected accesses shown in blue:
$\stackrel{1}{0} \rightarrow 2 \rightarrow 6 \rightarrow 3 \rightarrow 1 \rightarrow 4 \rightarrow 7 \rightarrow 5 \rightarrow 6 \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow 4 \rightarrow 6 \rightarrow 0 \rightarrow 7 \rightarrow$ $5 \rightarrow 1 \rightarrow \mathbf{0} \rightarrow 2 \rightarrow 7 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 6 \rightarrow 7 \rightarrow 0 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 3$

Note that this is one of many possible such sequences.

## 8 Emerging Memory Technologies [30 points]

Computer scientists at ETH developed a new memory technology, ETH-RAM, which is non-volatile. The access latency of ETH-RAM is close to that of DRAM while it provides higher density compared to the latest DRAM technologies. ETH-RAM has one shortcoming, however: it has limited endurance, i.e., a memory cell stops functioning after $10^{6}$ writes are performed to the cell (known as cell wear-out).

A bright ETH student has built a computer system using 1 GB of ETH-RAM as main memory. ETH-RAM exploits a perfect wear-leveling mechanism, i.e., a mechanism that equally distributes the writes over all of the cells of the main memory.
(a) [15 points] This student is worried about the lifetime of the computer system she has built. She executes a test program that runs special instructions to bypass the cache hierarchy and repeatedly writes data into different words until all the ETH-RAM cells are worn-out (stop functioning) and the system becomes useless. The student's measurements show that ETH-RAM stops functioning (i.e., all its cells are worn-out) in one year ( 365 days). Assume the following:

- The processor is in-order and there is no memory-level parallelism.
- It takes 5 ns to send a memory request from the processor to the memory controller and it takes 28 ns to send the request from the memory controller to ETH-RAM.
- ETH-RAM is word-addressable. Thus, each write request writes 4 bytes to memory.

What is the write latency of ETH-RAM? Show your work.

```
\(t_{\text {wear_out }}=\frac{2^{30}}{2^{2}} \times 10^{6} \times\left(t_{\text {write_ }} M L C+5+28\right)\)
\(365 \times 24 \times 3600 \times 10^{9} \mathrm{~ns}=2^{28} \times 10^{6} \times\left(t_{\text {write__ }} M L C+33\right)\)
\(t_{\text {write_ }} M L C=\frac{365 \times 24 \times 3600 \times 10^{3}}{2^{28}}-33=84.5 \mathrm{~ns}\)
```


## Explanation:

- Each memory cell should receive $10^{6}$ writes.
- Since ETH-RAM is word addressable, the required amount of writes is equal to $\frac{2^{30}}{2^{2}} \times 10^{6}$ (there is no problem if 1 GB is assumed to be equal to $10^{9}$ bytes).
- The processor is in-order and there is no memory-level parallelism, so the total latency of each memory access is equal to $t_{\text {write__ }} M L C+5+28$.
(b) [15 points] ETH-RAM works in the multi-level cell (MLC) mode in which each memory cell stores 2 bits. The student decides to improve the lifetime of ETH-RAM cells by using the single-level cell (SLC) mode. When ETH-RAM is used in SLC mode, the lifetime of each cell improves by a factor of 10 and the write latency decreases by $70 \%$. What is the lifetime of the system using the SLC mode, if we repeat the experiment in part (a), with everything else remaining the same in the system? Show your work.
$t_{\text {wear_out }}=\frac{2^{29}}{2^{2}} \times 10^{7} \times(25.35+5+28) \times 10^{-9}$
$t_{\text {wear_}}{ }^{\text {out }}=78579686.3 \mathrm{~s}=2.49$ year


## Explanation:

- Each memory cell should receive $10 \times 10^{6}=10^{7}$ writes.
- The memory capacity is reduced by $50 \%$ since we are using SLC: Capacity $=2^{30} / 2=2^{29}$
- The required amount of writes is equal to $\frac{2^{29}}{2^{2}} \times 10^{7}$.
- The SLC write latency is $0.3 \times t_{\text {write_MLC }}: t_{\text {write_ }}$ SLC $=0.3 \times 84.5=25.35 \mathrm{~ns}$


## 9 Branch Prediction [70 points]

A processor implements an in-order pipeline with 12 stages. Each stage completes in a single cycle. The pipeline stalls on a conditional branch instruction until the condition of the branch is evaluated. However, you do not know at which stage the branch condition is evaluated. Please answer the following questions.
(a) [15 points] A program with 1000 dynamic instructions completes in 2211 cycles. If 200 of those instructions are conditional branches, at the end of which pipeline stage the branch instructions are resolved? (Assume that the pipeline does not stall for any other reason than the conditional branches (e.g., data dependencies) during the execution of that program.)

```
At the end of the 7th stage.
Explanation: Total cycles \(=12+1000+200 * X-1\)
\(2211=1011+200 * X\)
\(1400=200 * X\)
\(X=6\)
Each branch causes 6 idle cycles (bubbles), thus branches are resolved at the end of 7 th stage.
```

(b) In a new, higher-performance version of the processor, the architects implement a mysterious branch prediction mechanism to improve the performance of the processor. They keep the rest of the design exactly the same as before. The new design with the mysterious branch predictor completes the execution of the following code in 115 cycles.

```
MOV R1, #0 // R1 = 0
LOOP_1:
    BEQ R1, #5, LAST // Branch to LAST if R1 == 5
    ADD R1, R1, #1 // R1 = R1 + 1
    MOV R2, #0 // R2 = 0
LOOP_2:
    BEQ R2, #3, LOOP_1 // Branch to LOOP_1 if R2==3.
    ADD R2, R2, #1 // R2 = R2 + 1
    B LOOP_2 // Unconditional branch to LOOP_2
LAST:
    MOV R1, #1 // R1 = 0
```

Assume that the pipeline never stalls due to a data dependency. Based on the given information, determine which of the following branch prediction mechanisms could be the mysterious branch predictor implemented in the new version of the processor. For each branch prediction mechanism below, you should circle the configuration parameters that makes it match the performance of the mysterious branch predictor.
i) [15 points] Static Branch Predictor

Could this be the mysterious branch predictor?

```
YES
NO
```

If YES, for which configuration below is the answer YES? Pick an option for each configuration parameter.
i. Static Prediction Direction
Always taken Always not taken

Explain:
$Y E S$, if the static prediction direction is always not taken.
Explanation: Such a predictor makes 6 mispredictions, which is the number resulting in 115 cycles execution time for the above program.
ii) [15 points] Last Time Branch Predictor

Could this be the mysterious branch predictor?

## YES NO

If YES, for which configuration is the answer YES? Pick an option for each configuration parameter.
i. Initial Prediction Direction

Taken Not taken
ii. Local for each branch instruction (PC-based) or global (shared among all branches) history?

Local
Global

Explain:
NO.

Explanation: There is not a configuration for this branch predictor that results in 6 mispredictions for the above program.
iii) [10 points] Backward taken, Forward not taken (BTFN)

Could this be the mysterious branch predictor?
YES NO
Explain:
NO.
Explanation: BTFN predictor does not make exactly 6 mispredictions for the above program.
iv) [15 points] Two-bit Counter Based Prediction (using saturating arithmetic)

Could this be the mysterious branch predictor?

## YES

NO
If YES, for which configuration is the answer $Y E S$ ? Pick an option for each configuration parameter.
i. Initial Prediction Direction

$$
\frac{00 \text { (Strongly not taken) }}{10 \text { (Weakly taken) }} \quad \frac{01 \text { (Weakly not taken) }}{11 \text { (Strongly taken) }}
$$

ii. Local for each branch instruction (i.e., PC-based, without any interference between different branches) or global (i.e., a single counter shared among all branches) history?
Local

Explain:
$Y E S$, if local history registers with 00 or 01 initial values are used.
Explanation: Such a configuration yields 6 mispredictions, which results in 115 cycles execution time for the above program.

## 10 DRAM Scheduling and Latency [80 points]

You would like to understand the configuration of the DRAM subsystem of a computer using reverse engineering techniques. Your current knowledge of the particular DRAM subsystem is limited to the following information:

- The physical memory address is 16 bits.
- The DRAM subsystem consists of a single channel and 4 banks.
- The DRAM is byte-addressable.
- The most-significant 2 bits of the physical memory address determine the bank.
- The DRAM command bus operates at 500 MHz frequency.
- The memory controller issues commands to the DRAM in such a way that no command for servicing a later request is issued before issuing a READ command for the current request, which is the oldest request in the request buffer. For example, if there are requests A and B in the request buffer, where A is the older request and the two requests are to different banks, the memory controller does not issue an ACTIVATE command to the bank that B is going to access before issuing a READ command to the bank that A is accessing.

You realize that you can observe the memory requests that are waiting to be serviced in the request buffer. At a particular point of time, you take the snapshot of the request buffer and you observe the following requests in the request buffer.

Requests in the request buffer (in descending order of request age, where the oldest request is on the top):
Read $0 \times 4 \mathrm{C} 80$
Read $0 \times 0140$
Read $0 \times 4 \mathrm{EC0}$
Read $0 \times 8000$
Read $0 \times 5000$
Read $0 \times 803 \mathrm{~F}$
Read $0 \times 4 \mathrm{E} 80$

At the same time you take the snapshot of the request buffer, you start probing the DRAM command bus. You observe the DRAM command type and the cycle (relative to the first command) at which the command is seen on the DRAM command bus. The following are the DRAM commands you observe on the DRAM bus while the requests above are serviced.

```
Cycle 0 --- PRECHARGE
Cycle 6 --- ACTIVATE
Cycle 10 --- READ
Cycle 11 --- READ
Cycle 21 --- PRECHARGE
Cycle 27 --- ACTIVATE
Cycle 31 --- READ
Cycle 32 --- ACTIVATE
Cycle 36 --- READ
Cycle 37 --- READ
Cycle 38 --- READ
Cycle 42 --- PRECHARGE
Cycle 48 --- ACTIVATE
Cycle 52 --- READ
```

Answer the following questions using the information provided above.
(a) [15 points] What are the following DRAM timing parameters used by the memory controller, in terms of nanoseconds?
i) ACTIVATE-to-READ latency

8 ns.

Explanation. After issuing the ACTIVATE command at cycle 6 , the memory controller waits until cycle 10 , which indicates that the ACTIVATE-to-READ latency is 4 cycles. The command bus operates at 500 MHz , so it has 2 ns clock period. Thus, the ACTIVATE-to-READ is $4 * 2=8 \mathrm{~ns}$.

30 ns .

Explanation. The bank activated at cycle 6 is precharged at cycle 21. Although the memory controller is idle after cycle 11, it waits until cycle 21, which means the ACTIVATE-to-PRECHARGE latency restricts the memory controller from issuing the PRECHARGE command earlier. Thus, the ACTIVATE-to-PRECHARGE latency is 15 cycles $=30 \mathrm{~ns}$.

## 12 ns.

Explanation. The PRECHARGE-to-ACTIVATE latency can be easily seen in the first two commands at cycles 0 and 6 . The PRECHARGE-to-ACTIVATE latency is 6 cycles $=12 \mathrm{~ns}$.
(b) [20 points] What is the row size in bytes? Explain your answer.

64 bytes.
Explanation. The Read request to address 0x803F (to Bank 2) does not require an ACTIVATE command, which means there is a row hit for that access. The open row was activated by the command issued for the request to the address 0 x 8000 . That means the target rows of both requests should be the same. When we look at the binary form of those addresses, we see that the least significant 6 bits are different ( 000000 for $0 x 8000$ and 111111 for 0 x 803 F ). That means at least 6 of the least significant bits should be the column bits.
Later, when we look at the commands issued for the requests to 0 x 4 ECO and 0 x 4 E 80 , we see that for both of those requests, the memory controller has opened a new row. Thus, the target rows of those requests should be different. Since only the 6 th bit (assuming the least significant bit is the 0th bit) is different between the two addresses, the 6 th bit should be part of the row address. So, of the 6 th bit is part of the row address, the number of columns bits should be 6 or less. As we previously found from the requests to $0 \times 8000$ and $0 \times 803 \mathrm{~F}$ that the number of column bits should be at least 6 , combining those two findings we can say that the number of column bits should be exactly 6 . Thus, the row size is $2^{6}=64$ bytes.
(c) [20 points] What is the status of the banks prior to the execution of any of the the above requests? In other words, which rows from which banks were open immediately prior to issuing the DRAM commands listed above? Fill in the table below indicating whether a bank has an open row, and if there is an open row, specify its address. If there is not enough information to infer the open row address, write unknown.

|  | Open or Closed? | Open Row Address |
| :---: | :---: | :---: |
| Bank 0 | Open | 5 |
| Bank 1 | Open | Unknown |
| Bank 2 | Closed | - |
| Bank 3 | Open | 192 |

Explanation. By decoding the accessed addresses we can find which bank and row each access targets. Looking at the commands issued for those requests, we can determine which requests needed PRECHARGE (row buffer conflict, the initially open row is unknown in this case), ACTIVATE (the bank is initially closed), or directly READ (the bank is initially open and the open row is the same as the one that the request targets).

| $\stackrel{\ddot{Z}}{\square}$ | 0x4C80 $\rightarrow$ Bank: | 1, Row: | 50 (PRECHARGE first. Any row other than 50 could be opened.) |
| :---: | :---: | :---: | :---: |
|  | 0x0140 $\rightarrow$ Bank: | 0, Row: | 5 (Row hit, so Bank 0 must have row 5 open.) |
|  | 0x4EC0 $\rightarrow$ Bank: | 1, Row: | 59 |
|  | 0x8000 $\rightarrow$ Bank: | 2, Row: | 0 (ACTIVATE is issued first. That means the bank was already closed.) |
|  | 0xF000 $\rightarrow$ Bank: | 3, Row: | 192 (Row hit, so Bank 3 must have row 192 open.) |
|  | 0x803F $\rightarrow$ Bank: | 2, Row: | 0 |
|  | 0x4E80 $\rightarrow$ Bank: | 1, Row: | 58 |

(d) [25 points] To improve performance, you decide to implement the idea of Tiered-Latency DRAM (TLDRAM) in the DRAM chip. Assume that a bank consists of a single subarray. With TL-DRAM, an entire bank is divided into a near-segment and far-segment. When accessing a row in the near-segment, the ACTIVATE-to-READ latency reduces by 2 cycles and the ACTIVATE-to-PRECHARGE latency reduces by 5 cycles. When accessing a row in the far-segment, the ACTIVATE-to-READ latency increases by 1 cycle and the ACTIVATE-to-PRECHARGE latency increases by 2 cycles.
Assume that the rows in the near-segment have smaller row ids compared to the rows in the far-segment. In other words, physical memory row addresses 0 through $N-1$ are the near-segment rows, and physical memory row addresses $N$ through $M-1$ are the far-segment rows.
If the above DRAM commands are issued 5 cycles faster with TL-DRAM compared to the baseline (the last command is issued in cycle 47), how many rows are in the near-segment? Show your work.

```
59 rows have to be in the near segment.
Explanation. There should 59 rows in the near-segment (rows 0 to 58) since rows until row id
5 8 \text { need to be accessed with low latency to get 5 cycle reduction. Rows 59 and 192 are in the}
far-segment, thus latency for accessing them increases slightly.
Here is the new command trace:
Cycle 0 -- PRECHARGE - Bank 1
Cycle 6 -- ACTIVATE - Bank 1, Row 50, near segment
Cycle 8 -- READ - Bank 1
Cycle 9 -- READ - Bank 0
Cycle 16 -- PRECHARGE - Bank 1
Cycle 22 -- ACTIVATE - Bank 1, Row 59, far segment
Cycle 27 -- READ - Bank 1
Cycle 28 -- ACTIVATE - Bank 2, Row 0
Cycle 30 -- READ - Bank 2
Cycle 31 -- READ - Bank 3
Cycle 32 -- READ - Bank 2
Cycle 39 -- PRECHARGE - Bank 1
Cycle 45 -- ACTIVATE - Bank 1, Row 58, near segment
Cycle 47 -- READ - Bank 1
```

