Midterm Exam

Computer Architecture (263-2210-00L)

ETH Zürich, Fall 2019

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Problem 1 (80 Points): RowHammer
Problem 2 (60 Points): DRAM Refresh
Problem 3 (60 Points): VRT and DRAM Refresh
Problem 4 (60 Points): In-DRAM Bit Serial Computation
Problem 5 (60 Points): Genome Analysis
Problem 6 (60 Points): Low-Latency DRAM

Total (380 Points):

Examination Rules:

1. Written exam, 180 minutes in total.
2. No books, no calculators, no computers or communication devices. 6 pages of handwritten notes are allowed.
3. Write all your answers on this document, space is reserved for your answers after each question. Blank pages are available at the end of the exam.
4. Clearly indicate your final answer for each problem. Answers will only be evaluated if they are readable.
5. Put your Student ID card visible on the desk during the exam.
6. If you feel disturbed, immediately call an assistant.
7. Write with a black or blue pen (no pencil, no green or red color).
8. Show all your work. For some questions, you may get partial credit even if the end result is wrong due to a calculation mistake. If you make assumptions, state your assumptions clearly and precisely.
9. Please write your initials at the top of every page.

Tips:

- Be cognizant of time. Do not spend too much time on one question.
- Be concise. You may be penalized for verbosity.
- Show work when needed. You will receive partial credit at the instructors’ discretion.
- Write legibly. Show your final answer.
1 RowHammer [80 points]

1.1 RowHammer Properties

Determine whether each of following statements is true or false. Note: we will subtract 1 point for each incorrect answer. (The minimum score you can get for this question is 0 point.)

(a) [2 points] Cells in a DRAM with a smaller technology node are more vulnerable to RowHammer.

1. True 2. False

(b) [2 points] Cells which have shorter retention times are especially vulnerable to RowHammer.

1. True 2. False

(c) [2 points] The vulnerability of cells in a victim row to RowHammer depends on the data stored in the victim row.

1. True 2. False

(d) [2 points] The vulnerability of cells in a victim row to RowHammer depends on the data stored in the aggressor row.

1. True 2. False

(e) [2 points] RowHammer-induced errors are mostly repeatable.

1. True 2. False

1.2 RowHammer Attacks

In order to characterize the vulnerability of your DRAM device to RowHammer attacks, you must be able to induce RowHammer errors. Assume the following about the target system:

- The CPU has a single in-order processor, and does not implement virtual memory.
- The physical memory address is 16 bits.
- The DRAM subsystem consists of two channels, four banks per channel, and 64 rows per bank.
- The memory controller employs open-page policy.
- The DRAM modules and the memory controller do not employ any remapping or scrambling schemes for the physical address.
- All the cells in the DRAM subsystem are equally vulnerable to RowHammer-induced errors.

You implement codes based on instructions shown in Table 1.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Description</th>
<th>Functionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>B LABEL</td>
<td>Unconditional Branch</td>
<td>PC = LABEL</td>
</tr>
<tr>
<td>STORE IMM, Rs</td>
<td>Store word to memory</td>
<td>MEM[IMM] = Rs</td>
</tr>
<tr>
<td>CLFLUSH IMM</td>
<td>Cache line flush</td>
<td>Flush cache line containing IMM</td>
</tr>
</tbody>
</table>

Table 1: Instruction Descriptions.

(a) [10 points] You run Code 1 below, but you cannot observe any errors in the target system. You figured out that the number of activations is much lower than your expectation. Give reason(s) as to why Code 1 cannot introduce a sufficient amount of activations.

Code 1

1: LOOP:
2: STORE 0x8732, R0
3: CLFLUSH 0x8732
4: B LOOP
All of reads in Code 1 are to the same row in DRAM, and the memory controller minimizes the number of DRAM commands by opening and closing the row just once, while issuing many column reads.

(b) [20 points] You try Codes 2a, 2b, and 2c, but find that only one of them can induce RowHammer errors in your DRAM subsystem. Which code segment is the one that can induce RowHammer errors? Justify your answer.

Code 2a

Code 2b

Code 2c

1. In order to introduce enough activations, two STORE instructions should access different rows in the same bank.
2. Three code segments are identical except for the memory addresses, so we can assume that only one code segment has two STORE instructions whose destination addresses are assigned to the same bank (but different rows).
3. Since the DRAM subsystem 1) consists of 8 banks and 2) employs no address remapping/scrambling schemes, two addresses assigned the same bank should satisfy a condition $C$: they have at least three same bit values at the same position.

Two addresses in each code are:

- **Code 2a**
  
  $1000 \ 0111 \ 0011 \ 0010 \ (0x8732)$
  $1001 \ 1000 \ 1100 \ 1101 \ (0x98CD)$

- **Code 2b**
  
  $1111 \ 0001 \ 1010 \ 1011 \ (0xF1AB)$
  $0000 \ 0000 \ 0101 \ 0100 \ (0x0054)$

- **Code 2c**
  
  $0010 \ 1011 \ 1001 \ 0111 \ (0x2B97)$
  $1101 \ 1010 \ 1110 \ 1000 \ (0xDA68)$

We can observe

1. Two paired addresses in every code segment have only three same bit values at the same position, i.e., satisfy $C$.
2. The position of the same bit values in Code 2b and Code 2c is the same, but different from Code 1. Therefore, if Code 2b can induce RowHammer errors, Code 2c should also be able to induce errors and Code 2a should not. On the other hand, if Code 2a can induce RowHammer errors, neither Code 2b or Code 2c can induce errors.

Since only one code segment can induce RowHammer errors, Code 2a is the one able to induce RowHammer errors.
1.3 RowHammer Mitigation Mechanisms

To identify a viable RowHammer mitigation mechanism for your system, you compare the two following mitigation mechanisms:

**Mechanism A.** The memory controller maintains a counter for every row, which increments every time the corresponding row is activated. If the counter value for a row exceeds a threshold value $T$, the memory controller activates the row’s two adjacent rows and resets the counter.

**Mechanism B.** Each time a row is closed (or precharged), the memory controller flips a biased coin with a probability $p$ of turning up heads, where $p << 1$. If the coin turns up heads, the memory controller activates one of its adjacent rows where either of the two adjacent rows are selected with equal probability ($p/2$).

(a) [5 points] You set $T$ for Mechanism A to 164 K based on the value of the Maximum Activation Count (MAC, i.e., the maximum number of times a row can be activated without inducing RowHammer errors in its adjacent rows) reported by the DRAM manufacturer. Calculate the number of bits required for counters in a memory controller which manages a single channel, 2 ranks per channel, 8 banks per rank, and $2^{15}$ rows per bank.

To count values up to 164 K, we need at least 18 bits ($2^{18} > 164K$) per counter.

\[ 18 \text{ [bit \ row]} \times (2^{15}) \text{ [row \ bank]} \times 16 \text{ [bank]} = 9 \times 2^{20} \text{ bits} = 9 \text{ Mib} \]

(b) [5 points] How does the answer to (a) change when both the number of rows per bank and the number of banks per chip are doubled?

It will increase to 36 Mib with 2x rows and 2x banks.

(c) [10 points] You profile the memory access pattern of the target system, and observe that the same pattern repeats exactly every 64 ms (the current refresh interval). Table 2 shows the number of activations for each row within a 64-ms time interval in a descending order. Given values $T = 164$ K for Mechanism A and $p = 0.001$ for Mechanism B, calculate the expected number of additional activations within a 64-ms time interval under each technique.

<table>
<thead>
<tr>
<th>Row Index</th>
<th># of ACTs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0x7332F</td>
<td>73 K</td>
</tr>
<tr>
<td>0x1802C</td>
<td>64 K</td>
</tr>
<tr>
<td>0x03F05</td>
<td>32 K</td>
</tr>
<tr>
<td>0x5FF02</td>
<td>10 K</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Total</td>
<td>480 K</td>
</tr>
</tbody>
</table>

Table 2: Number of Activations for Each Row.
Mechanism A introduces no additional row activation, since no row is activated more than the threshold.

On the other hand, for Mechanism B, the number of additional activations can be modeled as a random variable $X$ that is binomially-distributed with parameters $B(480,000, p)$.

$\therefore \# \ of \ additional \ activations = E(X) = 480,000 \times 0.001 = 480$

(d) [5 points] How does the answer to (c) change when both the number of rows per bank and the number of banks per chip are doubled? Assume that the memory access pattern does not change.

The performance overhead only depends on the access pattern, so it will not change.

(e) [10 points] What is the common challenge to implement the above mechanisms in the commodity systems?

This question is open ended. There could be other possible right answers.

Both Mechanisms require the information about exact mappings between row address and physical row, which is unlikely disclosed by manufacturers.

(f) [5 points] How can you address the common challenge?

This question is open ended. There could be other possible right answers.

Possible solutions
- Reverse engineering
- Implementing the techniques inside DRAM modules where the mapping function is managed.
- For counter-based approach, we can block future activations on a row (instead of refreshing its adjacent rows), if the counter value of a row exceeds the threshold value.
2 DRAM Refresh [60 points]

2.1 Basics [15 points]

A memory system is composed of eight banks, and each bank contains $2^{16}$ rows. Every DRAM row refresh is initiated by a command from the memory controller, and it refreshes a single row in a single DRAM bank. Each refresh command keeps the command bus busy for 5 ns. We define command bus utilization as the fraction of total execution time during which the command bus is occupied.

1. [5 points] Given that the refresh interval is 64 ms, calculate the command bus utilization of refresh commands. Show your work step-by-step.

Command bus is utilized for $8 \times 2^{16} \times 5 \text{ ns}$ at every 64 ms.

\[
Utilization = \frac{2^{19} \times 5 \text{ ns}}{2^6 \times 10^6 \text{ ns}} = \frac{2^{13}}{2 \times 10^5} = 2^{12} \times 10^{-5} = 4.096\
\]

Reduction = $1 - \frac{4.096}{2^{12} \times 10^{-5}} = 52.5\%$

2. [10 points] If 70% of all rows can withstand a refresh interval of 256 ms, how does the command bus utilization of refresh commands change? Calculate the reduction $(1 - \text{new/old})$ in bus utilization. Show your work step-by-step.

At every 256 ms:

- 70% of the rows are refreshed once.
  Command bus is busy for: $0.7 \times 8 \times 2^{16} \times 5 \text{ ns} = 3.5 \times 2^{19} \text{ ns}$

- 30% of the rows are refreshed four times.
  Command bus is busy for: $0.3 \times 8 \times 2^{16} \times 5 \text{ ns} \times 4 = 6 \times 2^{19} \text{ ns}$

\[
Utilization = (3.5 + 6) \times 2^{19} \text{ ns} / 256 \text{ ms} = 0.95 \times 2^{11} \times 10^{-5}
\]

\[
Reduction = 1 - \frac{0.95 \times 2^{11} \times 10^{-5}}{2^{12} \times 10^{-5}} = 52.5\%
\]

2.2 VRL: Variable Refresh Latency [45 points]

In this question, you are asked to evaluate "Variable Refresh Latency," proposed by Das et al. in DAC 2018.\(^1\)

The paper presents two key observations:

- First, a cell’s charge reaches 95% of the maximum charge level in 60% of the nominal latency value during a refresh operation. In other words, the last 40% of the refresh latency is spent to increase the charge of a cell from 95% to 100%. Based on this observation, the paper defines two types of refresh operations: (1) full refresh and (2) partial refresh. Full refresh uses the nominal latency value and restores the cell charge to 100%, while the latency of partial refresh is only 60% of the nominal latency value and it restores 95% of the charge.

• Second, a fully refreshed cell operates correctly even after multiple partial refreshes, but it needs to be fully refreshed again after a finite number of partial refreshes. The maximum number of partial refreshes before a full refresh is required varies from cell to cell.

The key idea of the paper is to apply a full refresh operation only when necessary and use partial refresh operations at all other times.

(a) [15 points] Consider a case in which:

• Each row must be refreshed every 64 ms. In other words, the refresh interval is 64 ms.
• Row refresh commands are evenly distributed across the refresh interval. In other words, all rows are refreshed exactly once in any given 64 ms time window.
• You are given the following plot, which shows the distribution of the maximum number of partial refreshes across all rows of a particular bank. For example, if the maximum number of refreshes is three, those rows can be partially refreshed for at most three refresh intervals, and the fourth refresh operation must be a full refresh.
• If all rows were always fully refreshed, the time that a bank is busy serving the refresh requests within a refresh interval would be \( T \).

How much time does it take (in terms of \( T \)) for a bank to refresh all rows within a refresh interval, after applying Variable Refresh Latency?

Full refresh latency = \( T \), partial refresh latency = 0.6T.

10% of the rows are fully refreshed at every other interval:
\[ 0.1 \times (1/2 \times 0.6T + 1/2 \times T) \]
40% of the rows are fully refreshed after every three partial refresh:
\[ 0.4 \times (3/4 \times 0.6T + 1/4 \times T) \]
30% of the rows are fully refreshed after every seven partial refresh:
\[ 0.3 \times (7/8 \times 0.6T + 1/8 \times T) \]
20% of the rows are fully refreshed after every fifteen partial refresh:
\[ 0.2 \times (15/16 \times 0.6T + 1/16 \times T) \]

Then, new refresh latency of a bank would be 0.68T.
(b) [15 points] You find out that you can relax the refresh interval, and define your baseline as follows:

- 75% of the rows are refreshed at every 128ms; 25% of the rows are refreshed at every 64ms.
- Refresh commands are evenly distributed in time.
- All rows are always fully refreshed.
- A single refresh command costs $0.2/N \text{ ms}$, where $N$ is the number of rows in a bank.
- Refresh overhead is defined as the fraction of time that a bank is busy, serving the refresh requests over a very large period of time.

Calculate the refresh overhead for the baseline.

At every 128ms:

25% of the rows are refreshed twice, 75% of the rows are refreshed once.

Total time spent for refresh in a 128 ms. interval is $(0.75N + 2 \times 0.25N) \times 0.2/N = 0.25ms$.

Then refresh overhead is $0.25/128$
(c) [15 points] Consider a case where:

- 75% of the rows are refreshed at every 128ms; 25% of the rows are refreshed at every 64ms.
- Refresh commands are evenly distributed in time.
- You are given the following plot, which shows the distribution of the maximum number of partial refreshes across all rows of a particular bank.
- A single refresh command costs \( 0.2/N \) ms, where N is the number of rows in a bank.
- Refresh overhead is defined as the fraction of time that a bank is busy, serving the refresh requests over a very large period of time.

![Plot showing distribution of maximum number of partial refreshes](image)

Calculate the refresh overhead. Show your work step-by-step. Then, compare it against the baseline configuration (the previous question). How much reduction \( (1 - \frac{\text{new}}{\text{old}}) \) do you see in the performance overhead of refreshes?

---

Full refresh of a row costs 0.2/N ms. Then, partial refresh of a row costs 0.12/N ms

**At every** 8 × 128 ms:

- 25% of the rows are refreshed for 8 times: 4 times **fully refreshed** and 4 times **partially refreshed**.
- 40% of the rows are refreshed for 8 times: 2 times **fully refreshed** and 6 times **partially refreshed**.
- 20% of the rows are refreshed for 16 times: 4 times **fully refreshed** and 12 times **partially refreshed**.
- 10% of the rows are refreshed for 8 times: 1 time **fully refreshed** and 7 times **partially refreshed**.
- 5% of the rows are refreshed for 16 times: 2 time **fully refreshed** and 14 times **partially refreshed**.

**Total time spent for refresh is:**

\[
\begin{align*}
&= (0.25N \times 4 + 0.4N \times 2 + 0.2N \times 4 + 0.1N \times 1 + 0.05N \times 2) \times 0.2/N \\
&+ (0.25N \times 4 + 0.4N \times 6 + 0.2N \times 12 + 0.1N \times 7 + 0.05N \times 14) \times 0.12/N \\
&= (1 + 0.8 + 0.8 + 0.1 + 0.1) \times 0.2 + (1 + 2.4 + 2.4 + 0.7 + 0.7) \times 0.12 \\
&= (0.56 + 0.864) = 1.424 \text{ ms}
\end{align*}
\]

Then, refresh overhead is 1.424/(8 × 128)

So, the reduction is 1 - (1.424/8)/0.25 ≈ 28.8%.
3  VRT and DRAM Refresh [60 points]

You observe that your system suffers from random bit flips in the main memory. All the observations suggest that the source of these bit errors is likely to be Variable Retention Time (VRT). Unfortunately, your hardware does not implement any ECC mechanism to correct those bit flips. You need a quick fix to mitigate these bit errors. Answer the questions for the following system configuration.

- The memory controller refreshes every DRAM row at every 64ms.
- The memory subsystem consists of one channel, one rank, and 16 banks.
- The total capacity of DRAM is 2GB. Each DRAM row contains 4kB. The cache line size is 64B.
- A DRAM cell that is vulnerable to VRT is called VRT cell regardless of its leakage state.
- VRT cells are uniformly distributed across the main memory.
- A VRT cell can retain its data for 128ms and 16ms in low- and high-leakage states, respectively.

(a) [15 points] Evaluate the idea of increasing the refresh rate. To avoid having VRT-related bit errors, refresh rate should support the worst retention time.

What should be the new refresh rate? How much time would refresh operations keep a bank busy in a 64ms of time window? Is increasing refresh rate a viable solution?

Each row should be refreshed at every 16ms instead of 64ms, which increases the bank utilization of refresh operations by 4x.

A bank spends 0.5ms for refreshing rows in a 64ms refresh window in baseline. Therefore, it spends 2ms for refreshing rows in a 64ms refresh window after increasing the refresh rate.

This solution could be viable as long as spending 2ms for refreshes does not hurt the workload performance, and the energy overhead of the increased refresh rate is not significant for the target use-case.
(b) [15 points] Your company also bought the newer generation of the same DRAM model with larger storage capacity that implements the same channel/rank/bank organization. Due to the higher capacity in the same chip area, the new generation employs smaller cells, which may have smaller retention time. Assume that:

- The retention time of a DRAM cell is halved for both low- and high-leakage states (i.e., 64ms in low-leakage and 8ms in high-leakage),
- The latency of refreshing one row remains the same.
- The capacity of new DRAM model is 64GB.

What should be the new refresh rate? How much time would refresh operations keep a bank busy in a 64ms of time window? Is increasing refresh rate a viable solution?

The retention time for the low-leakage state was 16ms. Now it is 8 ms. Therefore, we need to increase the refresh rate by 2x. The capacity increased from 2GB to 64GB while keeping the DRAM organization the same. Therefore the number of rows is 32x now, meaning that refresh cost should be 32x. Then, the new bank utilization is \(0.5 \times 2 \times 32 = 32\)ms.

Refreshing all rows in a bank takes 32ms, while each row needs to be refreshed every 8ms. Therefore, there is not enough time to refresh all rows. This solution is not viable.

(c) [30 points] Since this system is dedicated to run a particular workload, you need to analyze the workload access pattern to assess opportunities of having a low-cost solution. As the first step, you will consider a single VRT cell, which is located in Row A. Note that:

- The workload accesses to Row A exactly every 64ms, as shown at timestamps t1 and t3 below.
- The memory controller refreshes Row A every 64ms, as shown at timestamps t0 and t2 below.
- Refresh operations are not synchronized with the workload accesses. Assume that t2 can happen at any timestamp between t1 and t3 with equal probability.
- The cell changes its leakage state with a 30% of probability when the wordline is enabled.
- The cell initializes at low-leakage state, and gets immediately refreshed, as shown at timestamp t0 below.
- The cell retains its data for 128ms in low-leakage state and for 16ms in high-leakage state.

Calculate the probability of not violating the retention time of the cell for 64ms of execution. (Hint: You can create a tree for the probabilistic state transitions, as we partially provide below.)
We define $t_D$ as $t_1 - t_0$.

There are three paths to satisfy the retention time of the VRT cell until it reaches to $t_2$.

1. It remains in the low-leakage state during both the refresh at $t_0$ and the workload access at $t_1$. Probability of this case is $0.7 \times 0.7$.

2. It remains in the low-leakage state during the refresh at $t_0$, but switches to high-leakage during the workload access at $t_1$. In this case $t_2 - t_1$ should be smaller than 16ms to satisfy the retention time. Then, the probability of this case is $0.7 \times 0.3 \times P(t_D > 48ms)$

3. It switches to high-leakage state during the refresh at $t_0$, and switches back to low-leakage state during the workload access at $t_1$. In this case $t_1 - t_0$ should be smaller than 16ms to satisfy the retention time. Then, the probability of this case is $0.3 \times P(t_D < 16ms)$

The desired probability is the sum of these three probabilities.

Note that the retention time is inevitably violated in the fourth path, where the cell switches to high-leakage state during the refresh at $t_0$, and preserves its leakage state during the workload access at $t_1$. This happens because $t_D$ cannot be smaller than 16ms and larger than 48ms at the same time.
4 In-DRAM Bit Serial Computation [60 points]

Recall that in class, we discussed Ambit, which is a DRAM design that can greatly accelerate bulk bitwise operations by providing the ability to perform bitwise AND/OR of two rows in a subarray and NOT of one row. Since Ambit is logically complete, it is possible to implement any other logic gate (e.g., XOR). To be able to implement arithmetic operations, bit shifting is also necessary. There is no way of shifting bits in DRAM with a conventional layout, but it can be done with a bit-serial layout, as Figure 1 shows. With such a layout, it is possible to perform bit-serial arithmetic computations in Ambit.

We want to evaluate the potential performance benefits of using Ambit for arithmetic computations by implementing a simple workload, the element-wise addition of two arrays. Listing 1 shows a sequential code for the addition of two input arrays \( A \) and \( B \) into output array \( C \).

**Listing 1: Sequential CPU implementation of element-wise addition of arrays \( A \) and \( B \).**

```cpp
for (int i = 0; i < num_elements; i++){
    C[i] = A[i] + B[i];
}
```

We compare two possible implementations of the element-wise addition of two arrays: a CPU-based and an Ambit-based implementation. We make two assumptions. First, we use the most favorable layout for each implementation (i.e., conventional layout for CPU, and bit-serial layout for Ambit). Second, both implementations can operate on array elements of any size (i.e., bits/element):

- **CPU-based implementation**: This implementation reads elements of \( A \) and \( B \) from memory, adds them, and writes the resulting elements of \( C \) into memory.

  Since the computation is simple and regular, we can use a simple analytical performance model for the execution time of the CPU-based implementation: 
  \[ t_{cpu} = K \times num\_operations + \frac{num\_bytes}{M} \]
  where \( K \) represents the cost per arithmetic operation and \( M \) is the DRAM bandwidth.

- **Ambit-based implementation**: This implementation assumes a bit serial layout for arrays \( A \), \( B \), and \( C \). It performs additions in a bit serial manner, which only requires XOR, AND, and OR operations, as you can see in the 1-bit full adder in Figure 2.

  Ambit implements these operations by issuing back-to-back ACTIVATE (A) and PRECHARGE (P) operations. For example, to compute AND, OR, and XOR operations, Ambit issues the sequence of commands described in Table 3, where \( AAP(X,Y) \) represents double row activation of rows \( X \) and \( Y \) followed by a precharge operation, and \( AAAP(X,Y,Z) \) represents triple row activation of rows \( X \), \( Y \), and \( Z \) followed by a precharge operation.
In those instructions, Ambit copies the source rows $D_i$ and $D_j$ to auxiliary rows ($B_i$). Control rows $C_i$ dictate which operation (AND/OR) Ambit executes. The DRAM rows with dual-contact cells (i.e., rows $DCC_i$) are used to perform the bitwise NOT operation on the data stored in the row. Basically, the NOT operation copies a source row to $DCC_i$, flips all bits of the row, and stores the result in both the source row and $DCC_i$. Assume that:

- The DRAM row size is 8 Kbytes.
- An ACTIVATE command takes 20ns to execute.
- A PRECHARGE command takes 10ns to execute.
- DRAM has a single memory bank.
- The syntax of an Ambit operation is: $bbop_{[and/or/xor]} destination, source_1, source_2$.
- The rows at addresses $0x00700000$, $0x00800000$, and $0x00900000$ are used to store partial results. Initially, they contain all zeroes.
- The rows at addresses $0x00A00000$, $0x00B00000$, and $0x00C00000$ store arrays $A$, $B$, and $C$, respectively.
- These are all byte addresses. All these rows belong to the same DRAM subarray.

<table>
<thead>
<tr>
<th>$D_k = D_i$ AND $D_j$</th>
<th>$D_k = D_i$ OR $D_j$</th>
<th>$D_k = D_i$ XOR $D_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAP ($D_i, B_0$)</td>
<td>AAP ($D_j, B_1$)</td>
<td>AAP ($D_i, DCC_0$)</td>
</tr>
<tr>
<td>AAP ($D_j, B_0$)</td>
<td>AAP ($D_j, B_1$)</td>
<td>AAP ($D_j, DCC_1$)</td>
</tr>
<tr>
<td>AAP ($C_0, B_2$)</td>
<td>AAP ($C_1, B_2$)</td>
<td>AAP ($C_0, B_2$)</td>
</tr>
<tr>
<td>AAP ($B_0, B_1, B_2$)</td>
<td>AAP ($B_0, B_1, B_2$)</td>
<td>AAP ($B_0, DCC_1, B_2$)</td>
</tr>
<tr>
<td>AAP $B_0, D_k$</td>
<td>AAP $B_0, D_k$</td>
<td>AAP ($B_1, DCC_0, B_2$)</td>
</tr>
<tr>
<td>AAP ($C_1, B_2$)</td>
<td>AAP ($B_0, B_1, B_2$)</td>
<td>AAP ($C_1, B_2$)</td>
</tr>
<tr>
<td>AAP ($B_0, B_1, B_2$)</td>
<td>AAP ($B_0, B_1, B_2$)</td>
<td>AAP ($B_0, D_k$)</td>
</tr>
</tbody>
</table>

(a) [10 points] For the CPU-based implementation, you want to obtain $K$ and $M$. To this end, you run two experiments. In the first experiment, you run your CPU code for the element-wise array addition for 65,536 4-bit elements and measure $t_{cpu} = 100$ us. In the second experiment, you run the STREAM-Copy benchmark for 102,400 4-bit elements and measure $t_{cpu} = 10$ us. The STREAM-Copy benchmark simply copies the contents of one input array $A$ to an output array $B$. What are the values of $K$ and $M$?
\[ M = 10.24 \text{ GB/s} \text{ and } K = 1.38 \text{ ns/operation.} \]

**Explanation:**
We first calculate \( M \) by using the measurement for the STREAM-Copy benchmark, which does not involve any computation. For `num_bytes`, we count two arrays of 102,400 4-bit elements:
\[
t_{cpu} = \frac{\text{num\_bytes}}{M};
10 \times 10^{-6} = \frac{102,400 \times 4 \times 2}{8 \times M};
M = 10.24 \text{ GB/s}.\]

Then, we obtain \( K \) with the measurement for the array addition. For `num_operations`, we count the same number as `num_elements`. For `num_bytes`, we count three arrays of 65,536 4-bit elements:
\[
t_{cpu} = K \times \text{num\_operations} + \frac{\text{num\_bytes}}{M};
100 \times 10^{-6} = K \times 65,536 + \frac{65,536 \times 4 \times 3}{8 \times 10.24 \times 10^9};
K = 1.38 \text{ ns/operation.}\]

(b) [20 points] Write the code for the Ambit-based implementation of the element-wise addition of arrays \( A \) and \( B \). The resulting array is \( C \).

```c
// As we are doing bit serial computation, we need a for loop
// with as many iterations as the number of bits per element.
// We call \( n \) the number of bits per element.

for(int i = 0; i < n; i++){
    bbop_xor 0x00C00000+i*0x2000, 0x00A00000+i*0x2000, 0x00B00000+i*0x2000;
    bbop_and 0x00700000, 0x00A00000+i*0x2000, 0x00B00000+i*0x2000;
    bbcp_and 0x00800000, 0x00900000, 0x00C00000+i*0x2000;
    bbop_xor 0x00C00000+i*0x2000, 0x00900000, 0x00C00000+i*0x2000; // S
    bbop_or 0x00900000, 0x00700000, 0x00800000; // Cout
}
```
(c) [20 points] Compute the maximum throughput (in arithmetic operations per second, OPS) of the Ambit-based implementation as a function of the element size (i.e., bits/element).

\[
Thr_{ambit} = \frac{32}{n \times 10^{-9}} \text{ OPS} = \frac{32}{n} \text{ GOPS}
\]

**Explanation:**
Since DRAM has one single bank (and we can operate on a single subarray), the maximum throughput is achieved when we use complete rows. As the row size is 8KB, the maximum array size that we can work with is 65,536 elements.

First, we obtain the execution time as a function of the number of bits per element. Each XOR operation employs 25 ACTIVATION and 11 PRECHARGE operations. For AND and OR, 11 ACTIVATION and 5 PRECHARGE operations. Thus, the execution time of the bit serial computation on the entire array can be computed as (\(n\) is the number of bits per element):
\[
t_{ambit} = (2 \times t_{XOR} + 2 \times t_{AND} + t_{OR}) \times n;
\]
\[
t_{ambit} = 2030 \times n \text{ ns}.
\]

Second, we obtain the throughput in arithmetic operations per second (OPS) as:
\[
Thr_{ambit} = \frac{65,536}{2030 \times n \times 10^{-9}}; \quad Thr_{ambit} = \frac{32}{n} \text{ GOPS}.
\]

(d) [10 points] Determine the element size (in bits) for which the CPU-based implementation is faster than the Ambit-based implementation (Note: Use the same array size as in the previous part).

There is no number of bits per element that makes the CPU faster than Ambit.

**Explanation:**
We want to find \(n\) such that \(Thr_{ambit} < Thr_{cpu}\), or \(t_{ambit} > t_{cpu}\). If we use arrays of size 65,536 elements, we can write the following expression:
\[
t_{ambit} > t_{cpu};
\]
\[
2030 \times n \times 10^{-9} > 1.38 \times 65,536 \times 10^{-9} + \frac{65,536 \times 3 \times n}{8 \times 10^{24} \times 10^9}.
\]
This expression only returns a negative value of \(n\). Thus, there is no \(n\) that makes the CPU faster than Ambit.
5 Genome Analysis [60 points]

During a process called read mapping in genome analysis, each read (i.e., genomic subsequence) is mapped to one or more locations in the reference genome based on the similarity between the read and the reference genome segment at that location. Potential mapping locations are identified based on the presence of exact short segments (i.e., k-mers where $k$ is the length of the short segment) from the read sequence, in the reference genome. The locations of the k-mers in the reference genome are usually determined using a hash table. Each entry of the hash table stores a key-value pair, where the key is a k-mer and the value is a list of locations at which the k-mer occurs in the reference genome.

A challenge in designing such a hash table is deciding which k-mers to use as keys, as it affects the size of the hash table and the number of potential mapping locations, which affect the execution time of read mapping. In this question, you will be exploring the trade-offs between two strategies of k-mer selection:

(1) **Non-overlapping 4-mers**: Every non-overlapping 4-mers in the reference genome is used as a key in the hash table. For example, the reference AAAATTCA contains only two non-overlapping 4-mers: AAAA and TTCA. Thus, the hash table would have the following entries: \{AAAA\} \to \{1\} and \{TTCA\} \to \{5\}, where 1 and 5 are the start locations of the non-overlapping 4-mers (keys) in the reference.

(2) **Non-overlapping 4-mer minimizers**: For every non-overlapping 4-mer in the reference genome, the lexicographically minimum 4-mer of it and the two subsequent non-overlapping 4-mers is used as a key in the hash table. For example, the segment AAAATTCAACGGGCAG contains only two non-overlapping 4-mer minimizers, AAAA and ACGG. This is because AAAA is the lexicographically minimum k-mer among the first three consecutive k-mers (i.e., AAAA, TTCA, ACGG), and ACGG is the lexicographically minimum k-mer among the next three consecutive k-mers (i.e., TTCA, ACGG, and GCAG). Thus, the hash table would have the following entries: \{AAAA\} \to \{1\} and \{ACGG\} \to \{9\}, where 1 and 9 are the start locations of the minimizers (keys) in the reference.

Suppose that you would like to map a set of reads to the following reference genome. Note that the 4-mers are separated by ‘\_' only to help you identify the 4-mers easily, so you should not count them when creating a list of locations for a key.

AAAA_ATAC_TGAT_CCTT_ATAC_GTGG_TAAG_GTGT_ATAC_TAAG_TGAT

Answer the following questions based on the information given above:

(a) [10 points] Please list all \{key\} \to \{value\} entries in the hash table if we use all non-overlapping 4-mers as keys? The order of the entries is not important.

```
{AAAA} \to \{1\},
{ATAC} \to \{5, 17, 41\},
{TGAT} \to \{9, 49\},
{CCTT} \to \{13\},
{GTGG} \to \{21, 37\},
{TAAG} \to \{25, 45\},
{GTGT} \to \{29\},
{AAAA} \to \{33\}
```
(b) [10 points] Please list all \( \{\text{key}\} \rightarrow \{\text{value}\} \) entries in the hash table if we use all **non-overlapping 4-mer minimizers** as keys? Please list all the entries of this hash table. The order of the entries is **not** important.

\[
\begin{align*}
\{\text{AAAA}\} & \rightarrow \{1\}, \\
\{\text{ATAC}\} & \rightarrow \{5, 17, 41\}, \\
\{\text{GTTG}\} & \rightarrow \{21\}, \\
\{\text{CAAA}\} & \rightarrow \{33\}
\end{align*}
\]

(c) [20 points] Assume that we calculate the size of the hash table allocated in memory as: \(2^{\lceil \log_2 e \rceil} + p\) bytes, where \(e\) is the total number of hash table entries and \(p\) is the total number of locations stored across all values. Calculate the memory footprint (in bytes) of each of the two hash tables you designed in (a) and (b). Show your work.

Based on the content of the hash tables that we constructed in part a and part b of this question, we find that the size of the hash tables for:

1) Non-overlapping 4-mers: \(2^{\lceil \log_2 8 \rceil} + 13 = 21\) bytes
2) Non-overlapping 4-mer minimizers: \(2^{\lceil \log_2 4 \rceil} + 6 = 10\) bytes

Therefore the hash table with the non-overlapping 4-mers strategy requires \(\sim 2.1\times\) more memory than using the non-overlapping 4-mer minimizers. We also make the observation that it is **not** possible to add either 1) more key values (i.e., k-mers) or 2) more locations in the hash table using the non-overlapping 4-mer minimizers strategy than using the non-overlapping 4-mers as the latter already stores all possible non-overlapping \(\{\text{key}\} \rightarrow \{\text{value}\}\) pairs in the hash table.

Based on these observations and calculations, the non-overlapping 4-mer minimizers strategy consumes less memory as the \(k\) and \(p\) values cannot be larger than that of the non-overlapping 4-mers strategy.
(d) [20 points] Now assume we can query the hash table in $\log_2 e$ cycles, where $e$ is the total number of entries in the hash table. A read mapper queries the hash table using the first 4-mer of the read and calculates the edit distance between the read and the reference segment at each location returned by the hash table. Calculating the edit distance takes $l^2$ cycles where $l$ is the length of the read. If the edit distance between the read and a segment in the reference is higher than a certain threshold, the read mapper discards the location. We refer to cycles spent calculating the edit distance for segments at discarded locations as wasted cycles. When we profile read mapping with non-overlapping 4-mers and non-overlapping 4-mer minimizers strategies, we find the wasted cycles ratios to be 0.9 and 0.8, respectively. Assume that we want to align the read: GTTGACCAATGA to the reference genome above. What are the wasted cycles when aligning the read using 1) non-overlapping 4-mers and 2) non-overlapping 4-mer minimizers strategies? Please show your work.

We find from the question that each edit distance calculation takes $(\log_2 e) \ast n \ast (l^2)$ cycles where $n$ is the number of locations that the queried 4-mer exists in the reference genome, $(\log_2 e)$ is the cost to look up for the presence of the first 4 characters (i.e., 4-mer) of a read from the hash table, and $l^2$ is the cost of calculating the edit distance of a read. Based on the content of the hash tables that we constructed in part a and part b of this question, we find that:

- The number of keys in the hash tables are $e = 8$ and $e = 4$ when using 1) non-overlapping 4-mers and 2) non-overlapping 4-mer minimizers strategies, respectively.

- The size of the reads are 12 characters (i.e., $l = 12$)

- The first four characters of the read are: "GTTG". Thus, we find $n = 2$ locations (i.e., \{GTTG\} $\rightarrow$ \{21, 37\}) and $n = 1$ location (i.e., \{GTTG\} $\rightarrow$ \{21\}) when we query the hash tables that store 1) non-overlapping 4-mers and 2) non-overlapping 4-mer minimizers with "GTTG", respectively.

Based on these values we calculate the overall and wasted cycles cycles of both approaches as follows:

1) Non-overlapping 4-mers:

$Total \ cycles \ = \ (\log_2 8) \ + \ 2 \ast (12^2) \ = \ 291$

$Wasted \ cycles \ = \ Total \ cycles \ast 0.9 \ = \ 291 \ast 0.9 \ = \ 261.9$

2) Non-overlapping 4-mer minimizers:

$Total \ cycles \ = \ (\log_2 4) \ + \ (12^2) \ = \ 146$

$Wasted \ cycles \ = \ Total \ cycles \ast 0.8 \ = \ 146 \ast 0.8 \ = \ 116.8$
6 Low-Latency DRAM [60 points]

In class, we have seen the idea of Tiered-Latency DRAM (TL-DRAM). Recall that in TL-DRAM, each bitline of a subarray is segmented into two portions by adding isolation transistors in between, creating two segments on the bitline: the near segment and the far segment. The near segment is close to the sense amplifiers whereas the far segment is far away.

(a) [5 points] Why is accessing a row in the near segment faster in TL-DRAM compared to a commodity DRAM chip?

The bitline is shorter (thus lower capacitance) than commodity DRAM.

(b) [5 points] Why is accessing a row in the far segment slower in TL-DRAM compared to a commodity DRAM chip?

There is an additional capacitance because of the isolation transistor.

Now, assume that:

- We have a system that uses the near segment as a cache to the far segment, and the far segment contains main memory locations.
- The near segment is not visible to software and the rows that are cached in it are completely managed by the memory controller.
- The far segment is inclusive of the near segment.
- In each subarray, the far segment contains 496 rows whereas the total number of rows in the subarray is 512.

(c) [5 points] What is the capacity loss in main memory size when we use TL-DRAM as opposed to commodity DRAM with the same number of total DRAM rows? Express this as a fraction of total memory capacity lost (no need to simplify the fraction).

\[
\frac{16}{512}
\]
(d) [10 points] What is the tag store size that needs to be maintained on a per subarray basis in the DRAM controller if the near segment is used as a fully-associative write-back cache? Assume the replacement algorithm is Most Recently Used (MRU) and use the minimum number of bits possible to implement the replacement algorithm. Show your work.

\[
\text{180 bits}
\]

**Explanation:** 1 valid bit, 1 dirty bit, 9 bits tags, 16 different entries (one for each row)
Then we need 4 bits to point where the MRU entry is
Total size = 16*11 + 4 = 176 + 4 = 180 bits

Now assume near segment and far segment are exclusive. In other words, both contain memory rows, and a memory row can only be in one of the segments. When a memory row in the far segment is referenced, it is brought into the near segment by exchanging the MRU row in the near segment with the row in the far segment. Note that a row can end up in a different location in the far segment after being moved to the near segment and then back to the far segment.

(e) [5 points] When the near segment is used as an exclusive cache to the far segment, what is the capacity loss in main memory size when we use TL-DRAM as opposed to commodity DRAM with the same number of total DRAM rows? Express this as a fraction of total memory capacity lost (no need to simplify the fraction).

\[
\frac{1}{512} \text{ (or 0%)}
\]

**Explanation:** In this question, we accept 0% as an answer, but the correct answer is 1/512 Because you need a dummy row to perform the swap.

(f) [10 points] What is the tag store size that needs to be maintained on a per subarray basis in the DRAM controller if the near segment is used as an exclusive fully-associative write-back cache? Assume the replacement algorithm is MRU and use the minimum number of bits possible to implement the replacement algorithm. Show your work.

\[
\text{4612 bits}
\]

**Explanation:** Near segment: (9 bits tags) * 16row = 144 + 4 MRU bits = 148 bits
A row swap can lead to any physical pages in the subarray getting mapped to any row in the subarray. This means we need the tag bits in each of the far segment rows.
Far segment: 9 bits tags * 496 rows = 4464 bits
Total size = 4612 bits
(g) [20 points] Assume the near and far segments are visible to the operating system (OS) and the OS can allocate physical pages in either of the segments. Answer the following questions as True or False and provide explanation to your answer.

- The OS cannot manage the near segment as a cache at granularity smaller than the physical page granularity.

  True.
  **Explanation:**
  The statement is true as the smallest size that the OS can allocate from the physical memory is the page size of the system. Mapping virtual addresses to physical addresses at smaller chunks would require additional metadata to be stored in the page table, which is essentially equivalent of having a system with small pages.

- If the near segment is used as an OS-managed cache to store frequently-accessed pages, the cache can only be exclusive.

  False.
  **Explanation:**
  The OS-managed cache can either be exclusive or inclusive. For the exclusive option, the OS can swap a frequently-accessed page in the far segment with a page from the near segment. In contrast, to use the near segment as inclusive cache, the OS can copy a frequently-accessed page in the far segment to the near segment.

- There is zero memory capacity loss when the near segment is used as OS-managed cache.

  False.
  **Explanation:**
  The statement is incorrect when the OS uses the near segment as an inclusive cache since a virtual page can be mapped to a physical page from both near and far segments at the same time. In addition, the OS needs to reserve some memory capacity to maintain metadata for keeping track of the physical page from the far segment that a virtual page was mapped to before being copied to the near segment. Using the near segment as exclusive cache would also cause memory capacity loss due to 1) metadata that will be potentially needed for the page replacement policy and 2) an empty page that is needed to swap the data of a page from far and near segments.
• An OS-managed near segment cache does not incur any tag store overhead in the memory controller.

True.

**Explanation:** This statement is true assuming purely OS-managed cache design. However, it is also possible that the memory controller provides architectural support to the OS to store tags efficiently when using the near segment as an inclusive cache.