SMASH

Co-Designing Software Compression and Hardware-Accelerated Indexing for Efficient Sparse Matrix Operations

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Reviewed by Tuan Pham Huu

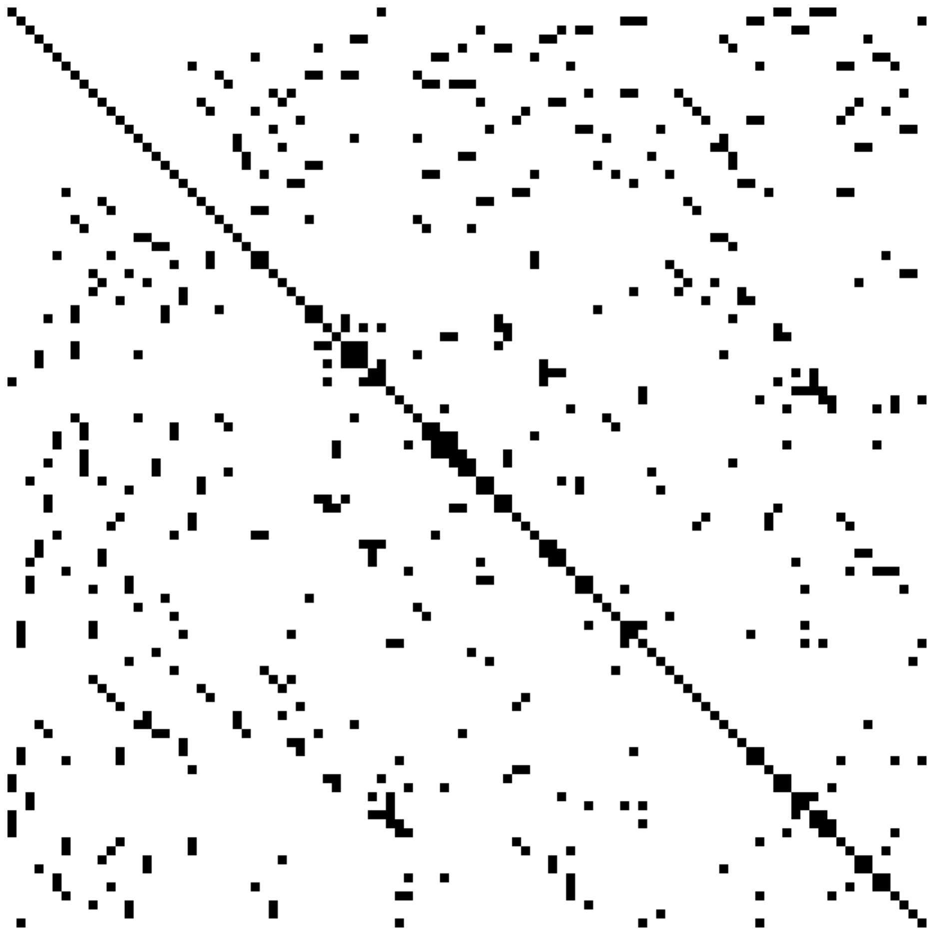
Executive Summary

- Many applications heavily rely on **sparse linear algebra**
- They require **effective compression formats** to mitigate **computational and storage overheads**
- Existing **formats** suffer from the **expensive cost** of finding the **position of non-zero elements**
- **SMASH**: **Hardware/Software cooperative mechanism** for **efficient non-zero elements discovery** and **sparse matrix operations**
  - **Software**: Efficient compression with a **hierarchy of bitmaps**
  - **Hardware**: Scans bitmaps to **find indices of non-zero elements**
- **38 - 44 %** faster than the state-of-the-art for **SpMV and SpMM**
- SMASH is **highly effective, low cost, and widely applicable**

Outline

- Definition and Applications of a **Sparse Matrix**
- Properties of an **Effective Compression Format**
- State-of-the-art: **CSR and BCSR**
- **Indexing Overhead** of CSR and BCSR
- **SMASH**: Effective Mechanism for Sparse Matrix Compression
- **Evaluation Methodology** and **Key Results**
- **Conclusion**
- **Critique** and **Discussion**

Definition: Sparse Matrix



- **Majority** of its values are 0
- **Widely used** in modern applications
- Vary in **size** and **sparsity**
- **Computations** with zeros are **unnecessary**
- **Storing** zeros is **wasteful** and/or **infeasible**

https://en.wikipedia.org/wiki/Sparse_matrix#/media/File:Finite_element_sparse_matrix.png

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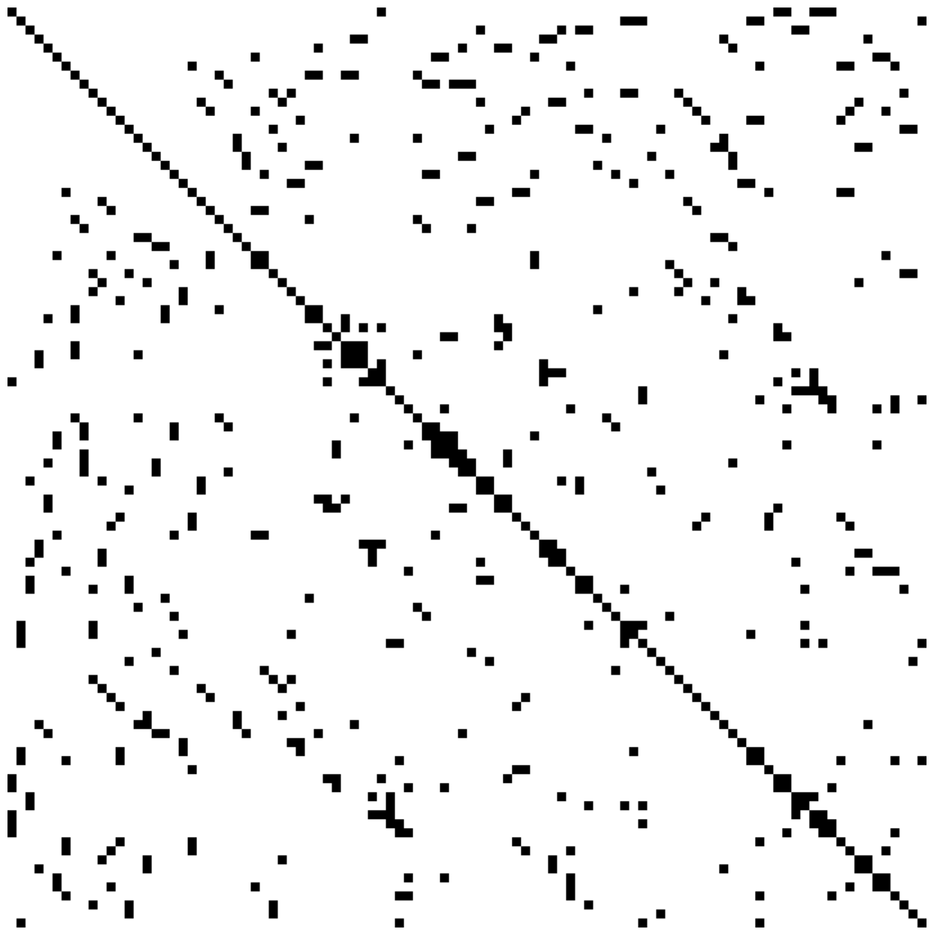
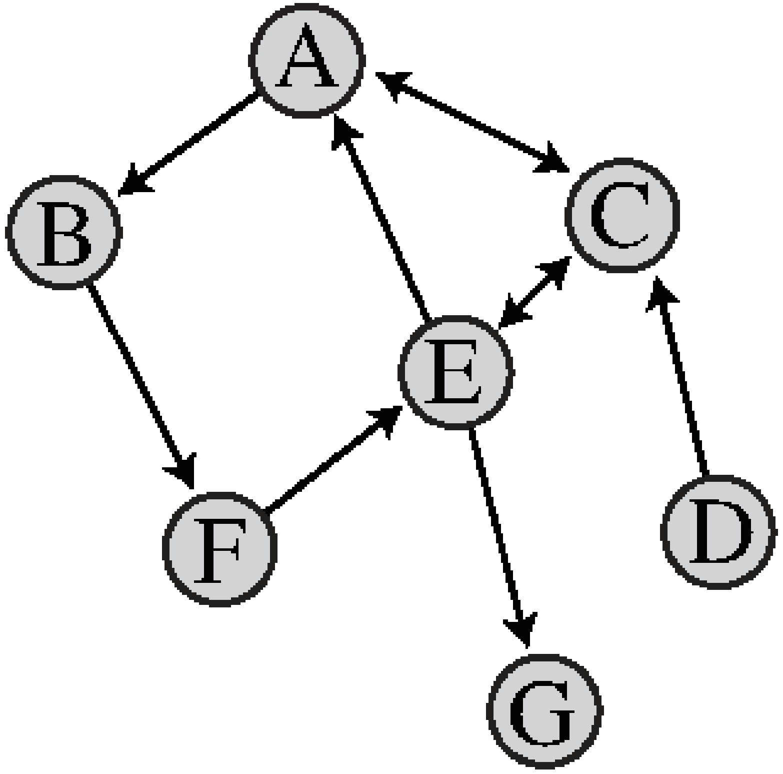


Need
sparse matrix compression
to avoid
overheads

Application: Graph Analytics For Social Networks



Adjacency Matrix M

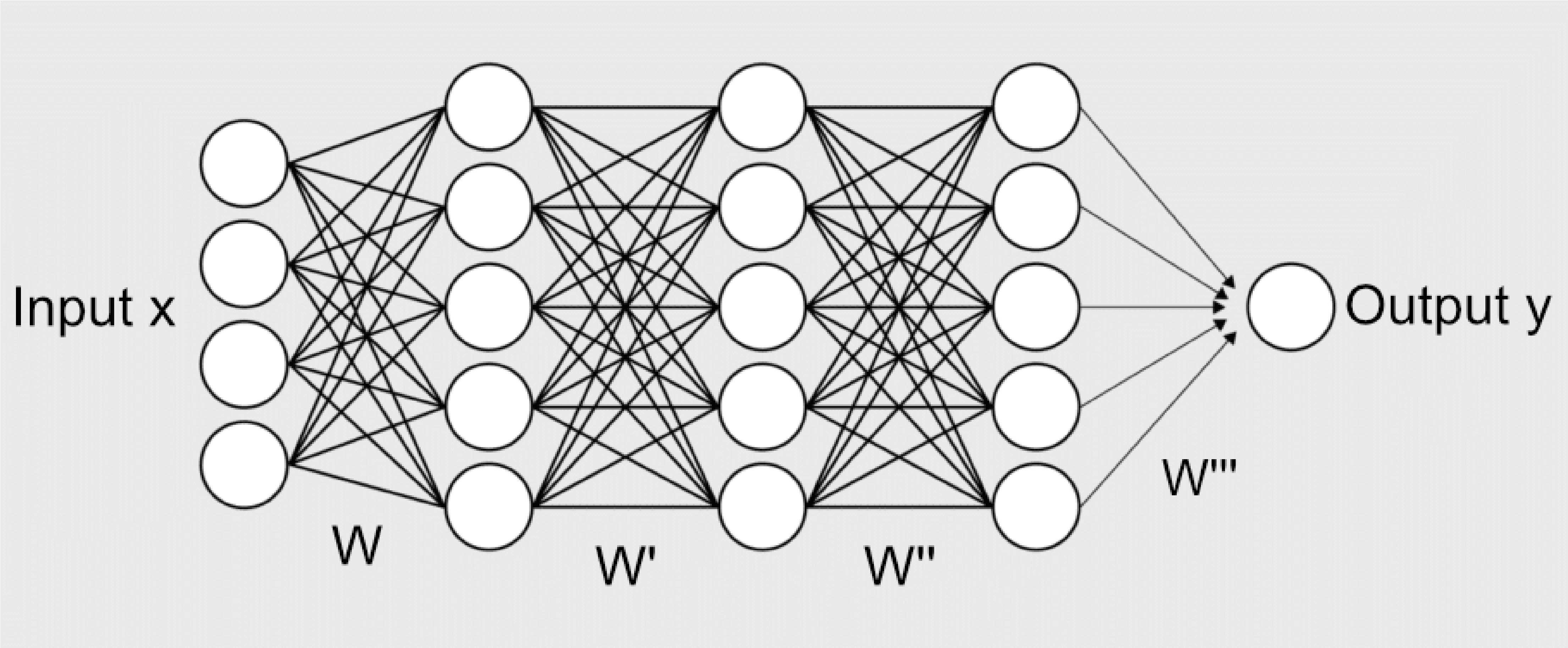
	A	B	C	D	E	F	G
A	0	1	1	0	0	0	0
B	0	0	0	0	0	1	0
C	1	0	0	0	1	0	0
D	0	0	1	0	0	0	0
E	1	0	1	0	0	0	1
F	0	0	0	0	1	0	0
G	0	0	0	0	0	0	0

https://www.semanticscholar.org/paper/Topics-in-social-network-analysis-and-network-O'Malley-Onnela/61ede53272f795e1115badd45f28164e34b98a37/figure/0

Application: Graph Analytics For Social Networks

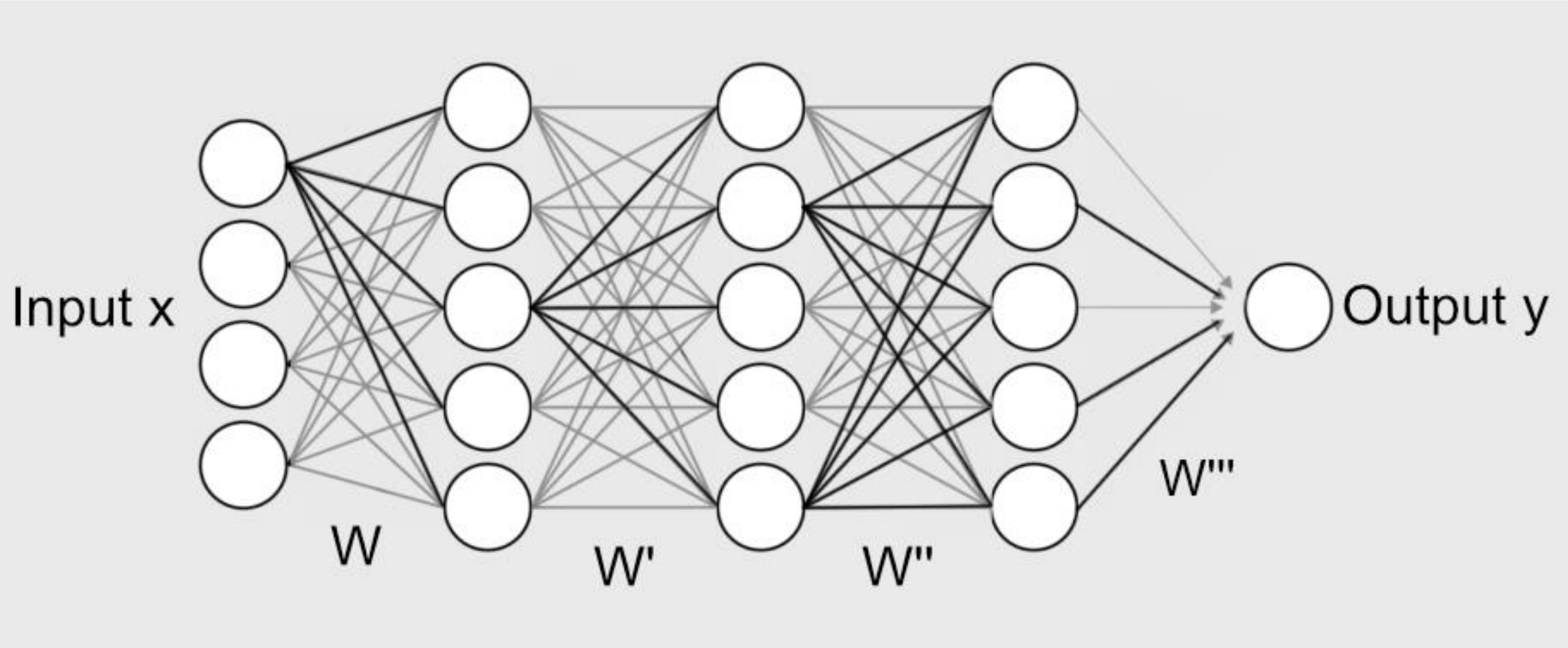
https://www.facebook.com/notes/facebook-engineering/visualizing-friendships/469716398919

Application: Sparse Weights in DNN with Dropout



https://dimensionless.in/what-is-neural-network/neural-network-schematic/

Application: Sparse Weights in DNN with Dropout

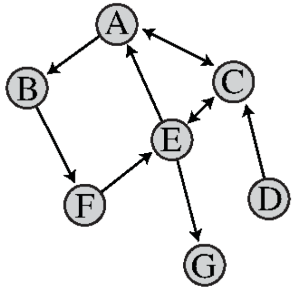


Application: Diagonal Matrix Computation

$$\underbrace{\begin{pmatrix} d_1 & 0 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 & 0 \\ 0 & 0 & d_3 & 0 & 0 \\ 0 & 0 & 0 & d_4 & 0 \\ 0 & 0 & 0 & 0 & d_5 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}}_{x} = diag(A) \underset{cwise}{*} x = \underbrace{\begin{pmatrix} d_1 * x_1 \\ d_2 * x_2 \\ d_3 * x_3 \\ d_4 * x_4 \\ d_5 * x_5 \end{pmatrix}}_{y}$$

Exploiting the structure of A → From quadratic to linear complexity

Properties of an Effective Compression Format



Adjacency Matrix M

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A	0	1	1	0	0	0	0
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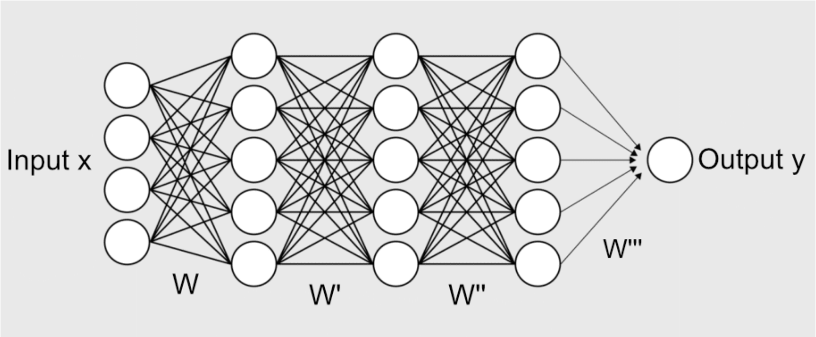
G	0	0	0	0	0	0	0

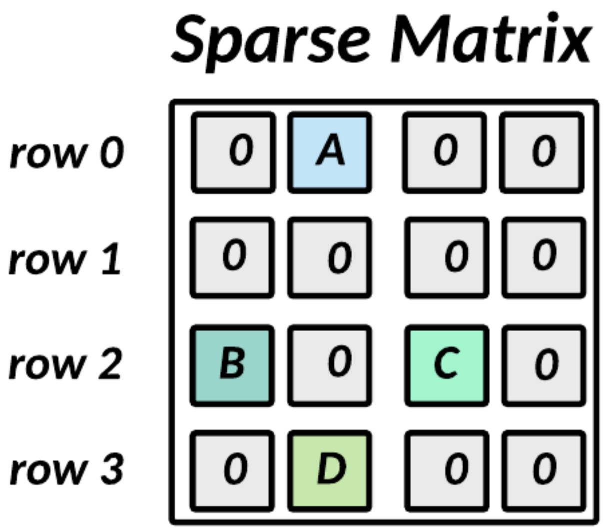


$$\underset{A}{\begin{pmatrix} d_1 & 0 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 & 0 \\ 0 & 0 & d_3 & 0 & 0 \\ 0 & 0 & 0 & d_4 & 0 \\ 0 & 0 & 0 & 0 & d_5 \end{pmatrix}} \underset{x}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}} = diag(A) \underset{cwise}{*} x = \underset{y}{\begin{pmatrix} d_1 * x_1 \\ d_2 * x_2 \\ d_3 * x_3 \\ d_4 * x_4 \\ d_5 * x_5 \end{pmatrix}}$$

1. **Storage Efficiency**
2. **Computational Efficiency**
3. **General Applicability**

Compressed Sparse Row (CSR)

- **Most widely** used (Intel MKL, OpenBLAS, Eigen, etc.)
- Compressed Sparse Column (**CSC**) follows the same concept



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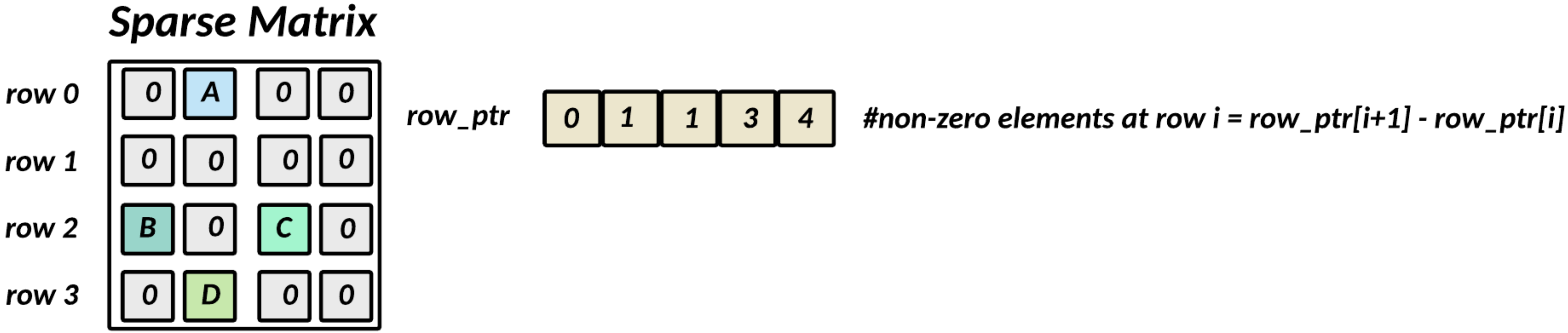

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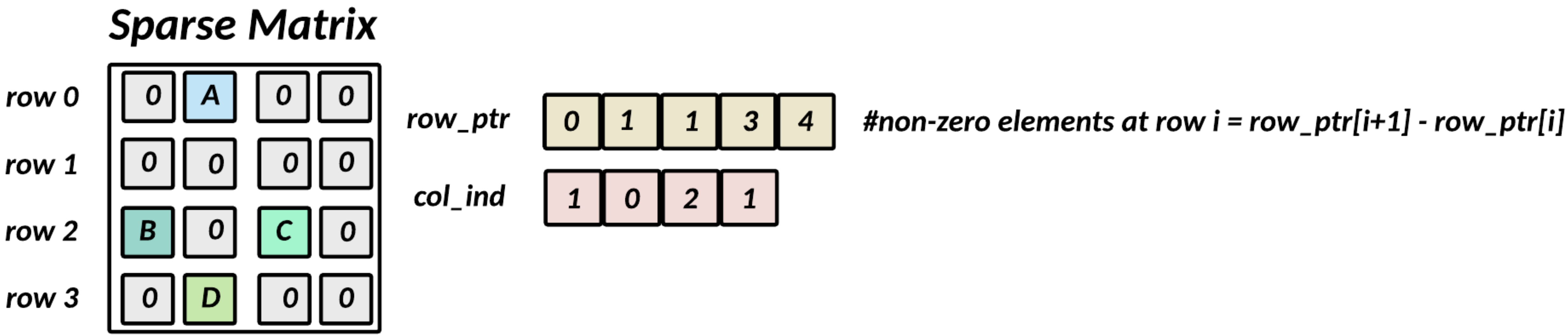

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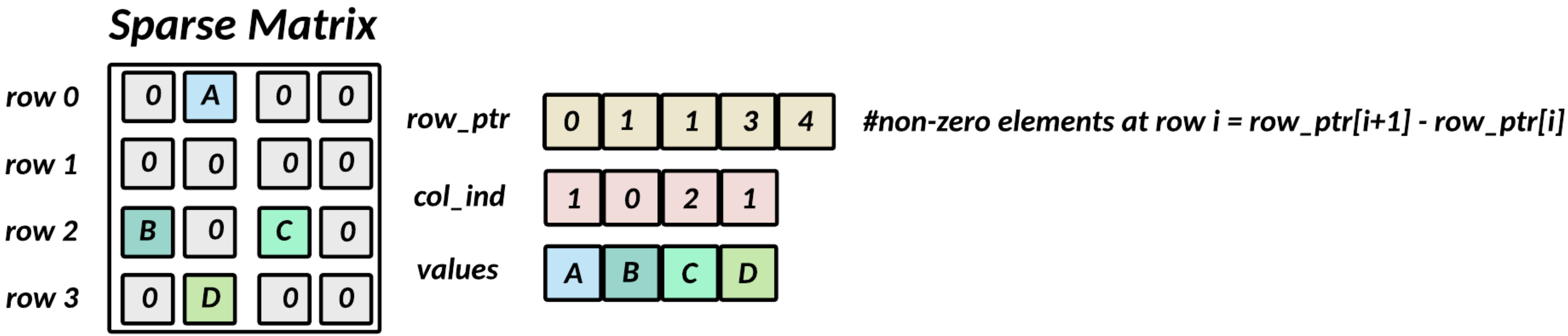


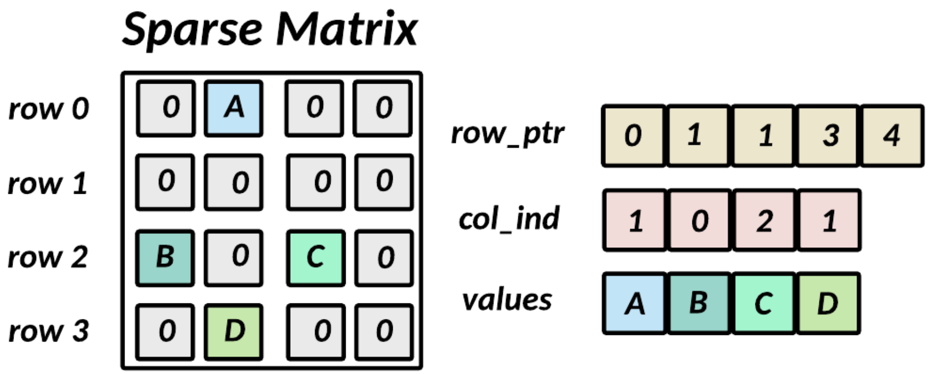
Compressed Sparse Row (CSR)

Advantages

- Enables **spatial locality**
- Only stores **non-zero** values (ignoring meta-data)

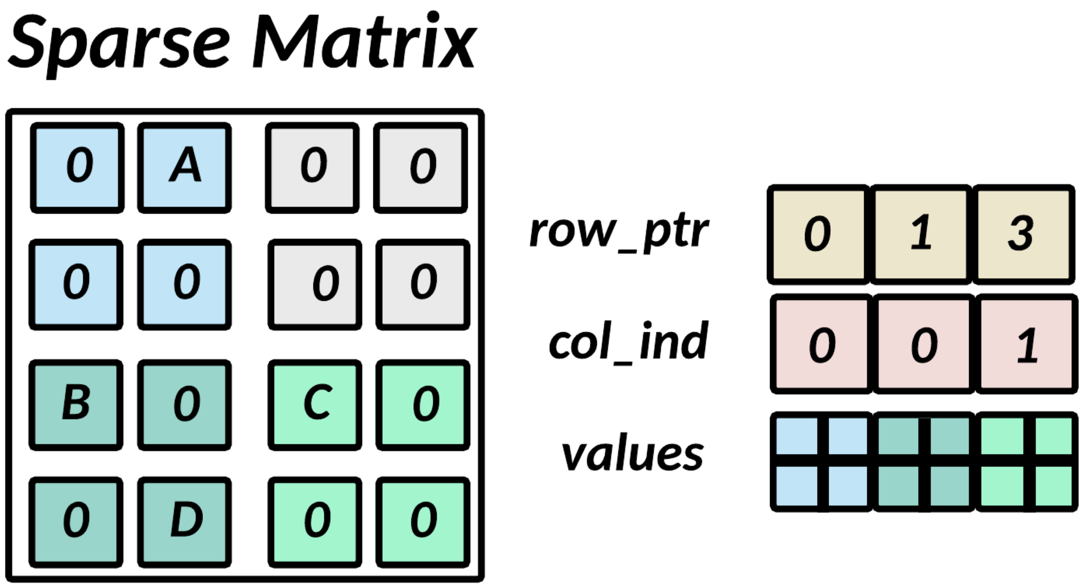
Disadvantages

- **Costly insertion** of non-zero values
- Difficult to obtain **temporal locality**



Block Compressed Sparse Row (BCSR)

- **Modification** of CSR



Block Compressed Sparse Row (BCSR)

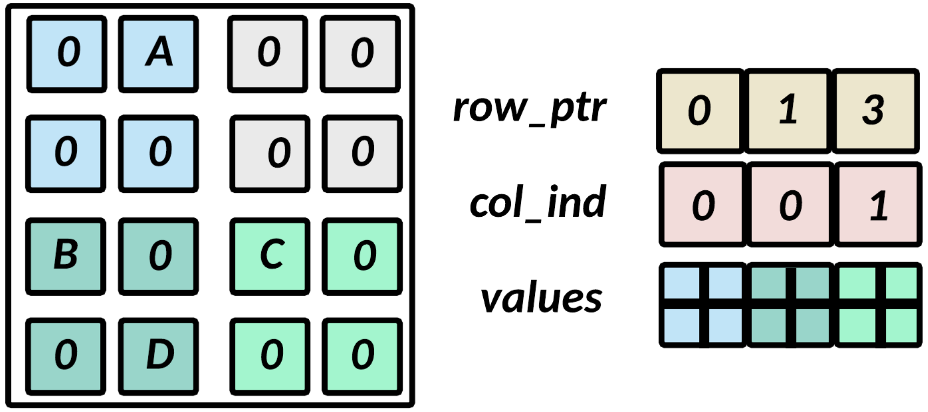
Advantages

- **Reduced storage** for indices
- Enables **blocking optimization** in matrix operations

Disadvantages

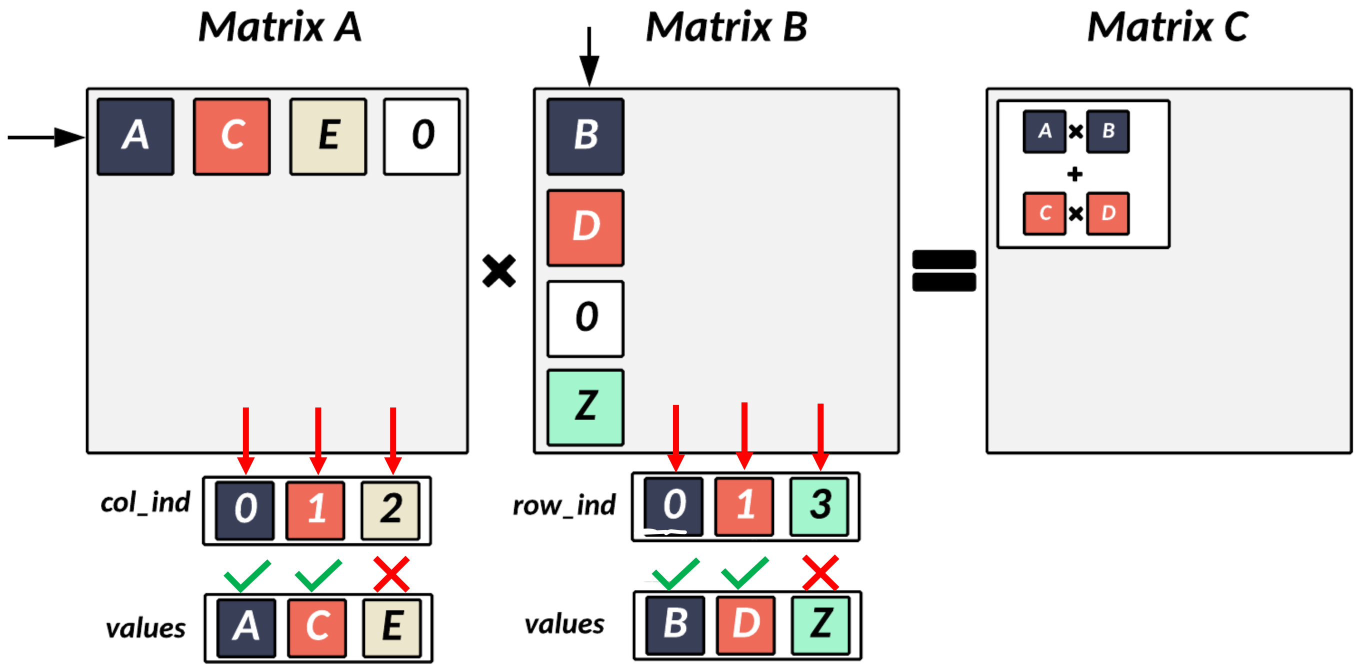
- Storage of **zeros**
- Computational **overhead**

Sparse Matrix



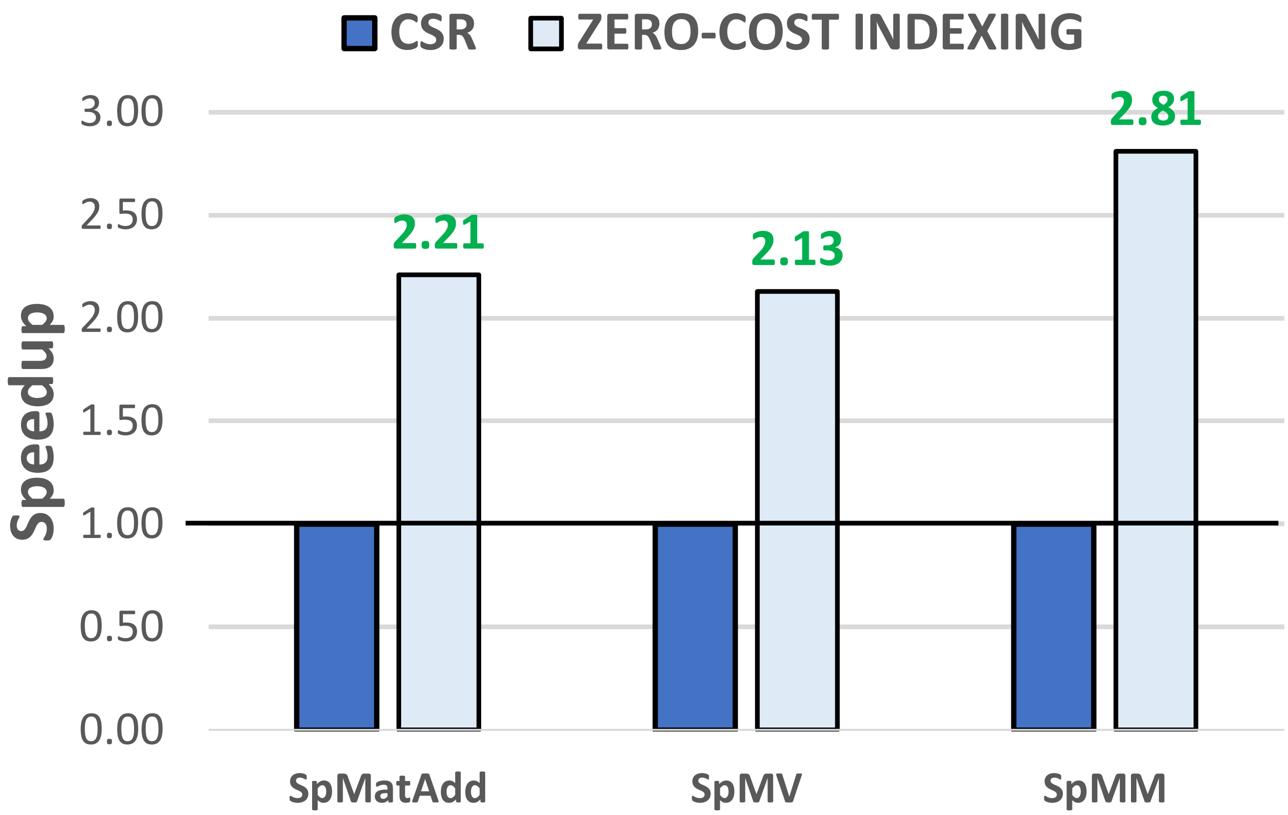
Indexing Overhead of CSR and BCSR

- **Multiple** memory instructions to get **position of non-zero value or block**
- **Comparison** of indices



Zero-cost Indexing for CSR and BCSR

Zero-cost indexing indicates room for improvement

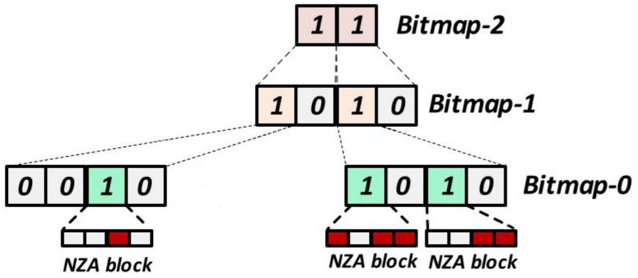


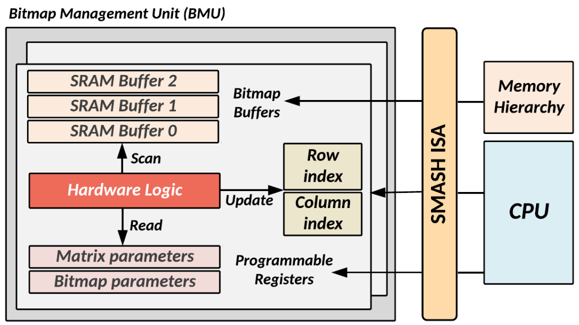
SMASH: A Novel Compression Approach

Hardware/Software Cooperative Mechanism

- Enables highly efficient sparse matrix compression and computation
- Works effectively for a diverse set of sparse matrices

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&0&1&1&0&0&0&0\\ 0&0&1&1&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$



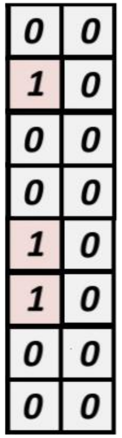


Software Compression

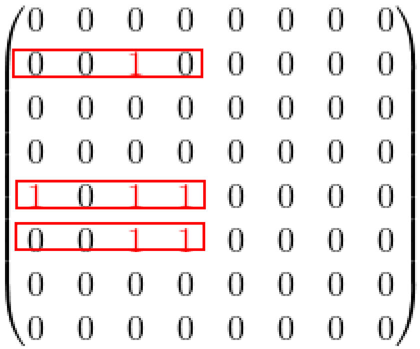
Hardware-accelerated Indexing

SMASH: Software Compression Scheme

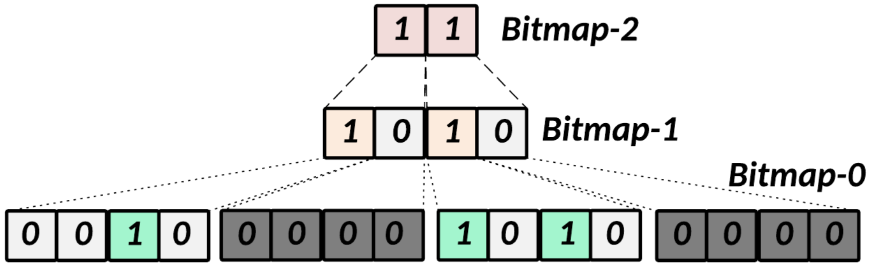
Single Bitmap



Non-zero blocks



Hierarchy of Bitmaps



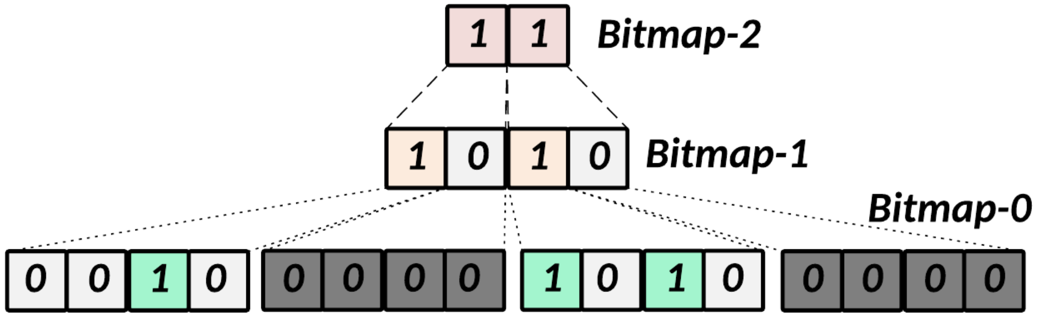
- Bit encodes the existence of a **non-zero** value in a region
- A lot of zero bits

- **Idea:** Apply encoding recursively to reduce the number of zero bits (**bottom-up**)

SMASH: Software Compression Scheme

Levels

- Indicate presence of non-zero values with **different granularity**



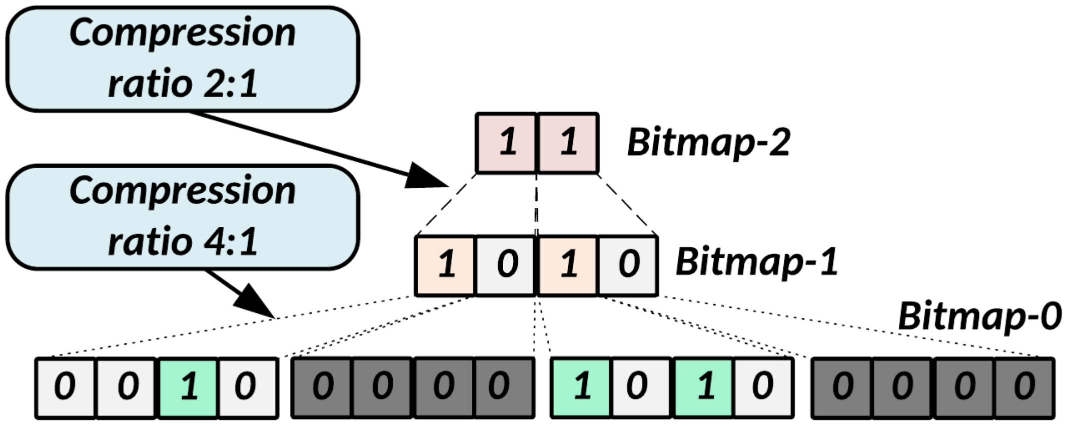
SMASH: Software Compression Scheme

Levels

- Indicate presence of non-zero values with **different granularity**

Compression Ratios

- **Configurable**, determine **change in granularity** between levels



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- Indicate presence of non-zero values with **different granularity**

Compression Ratios

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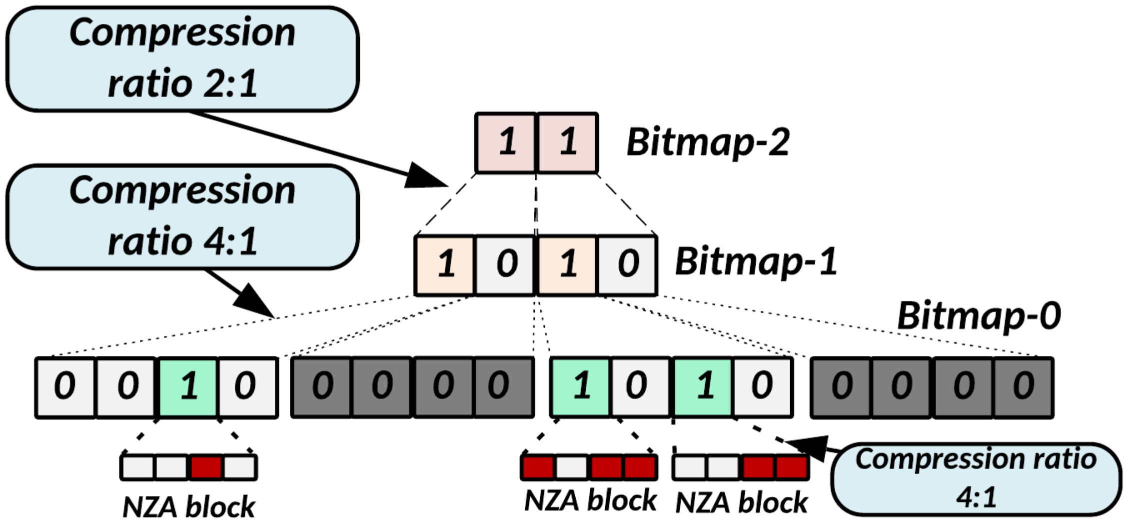
Non-zero Array (NZA)

- Stores **non-zero blocks** of matrix



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Levels

- Indicate presence of non-zero values with **different granularity**

Compression Ratios

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Non-zero Array (NZA)

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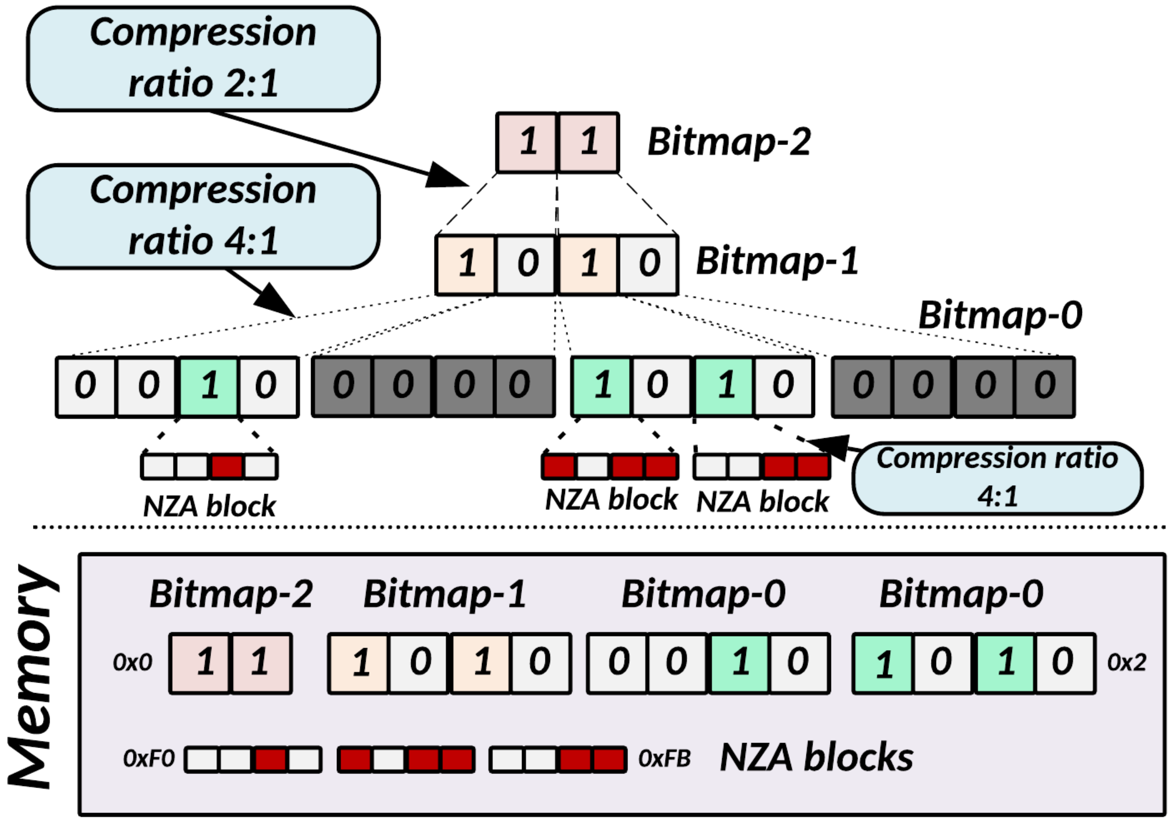
Representation in Memory

- Only store **necessary** information



SMASH: Software Compression Scheme

Efficient storage

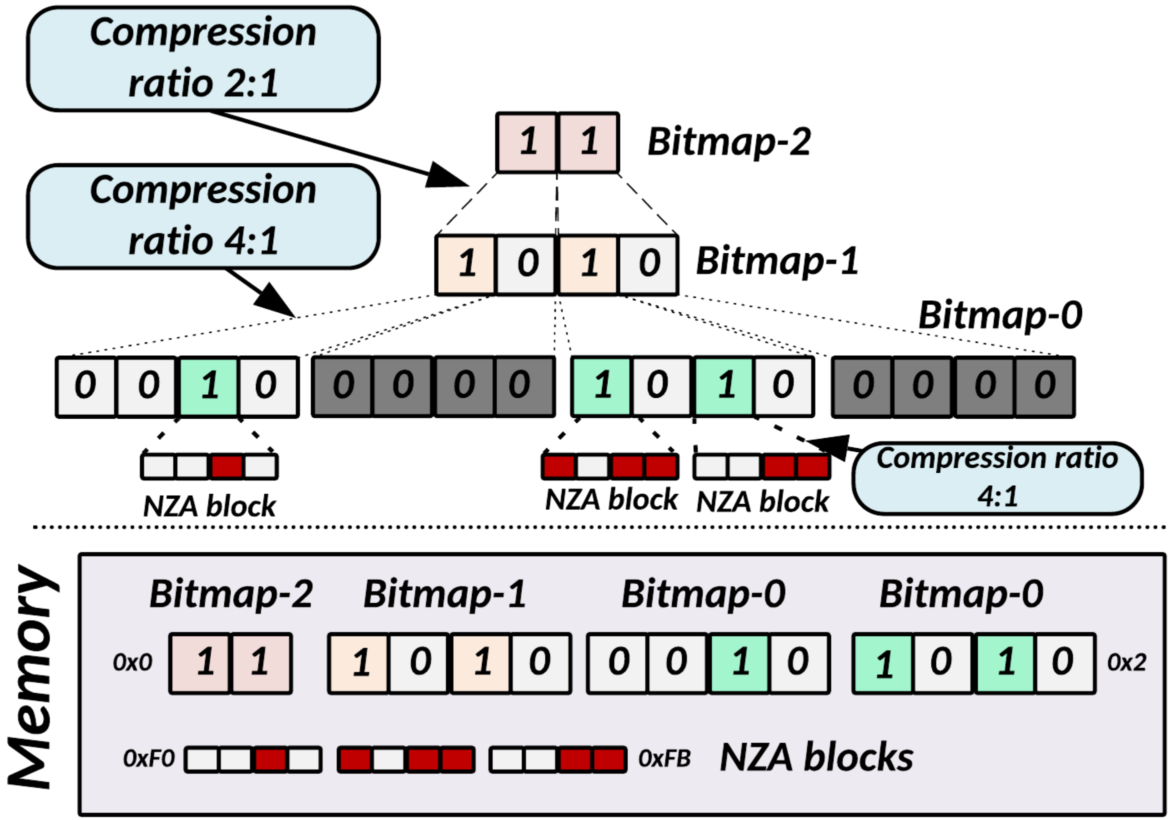
- Each non-zero block has only a few bits (= #levels) of meta-data

Fast indexing

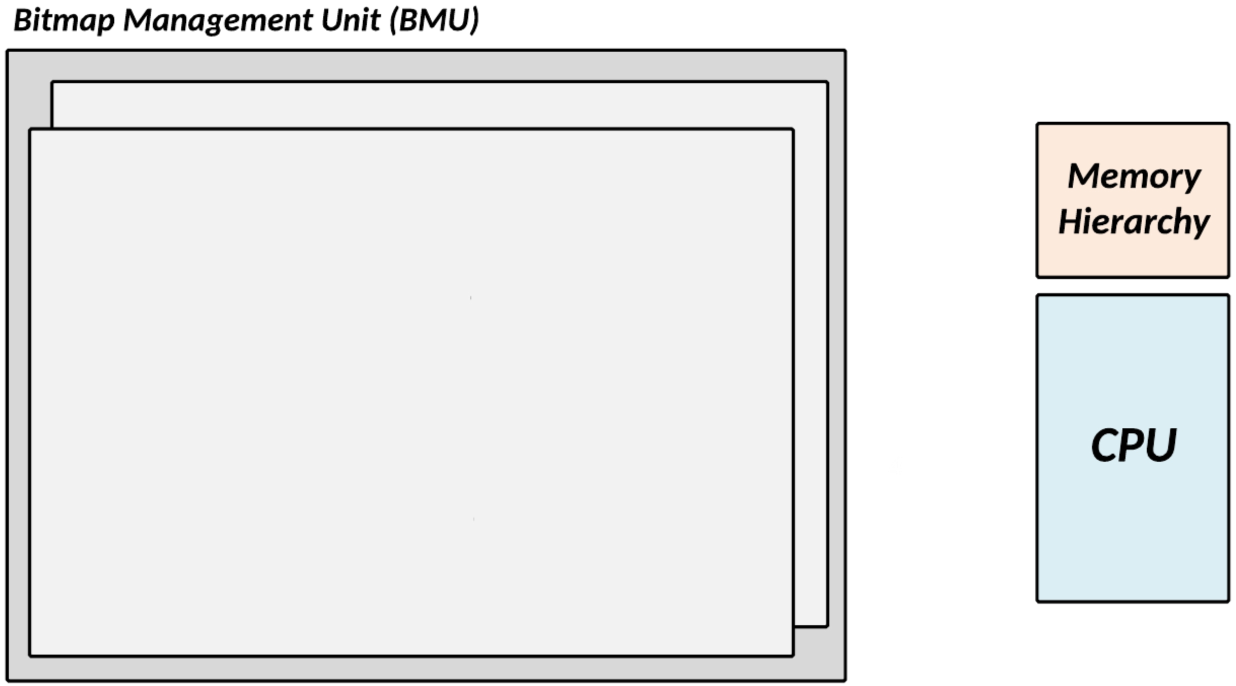
- Traverse bitmap hierarchy in a depth-first manner to compute the index
- Index is a simple sum of products

Enables hardware acceleration

- Little amount of data to transfer
- Facilitated by hierarchical structure

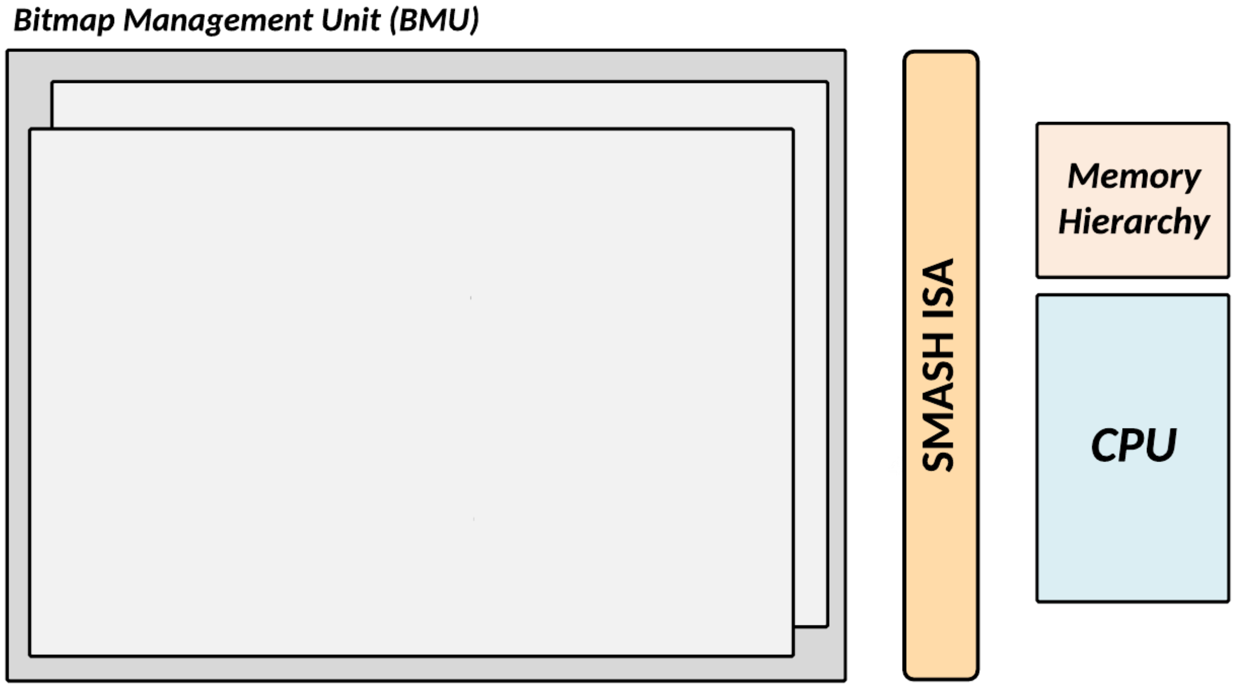


SMASH: Hardware Acceleration Scheme



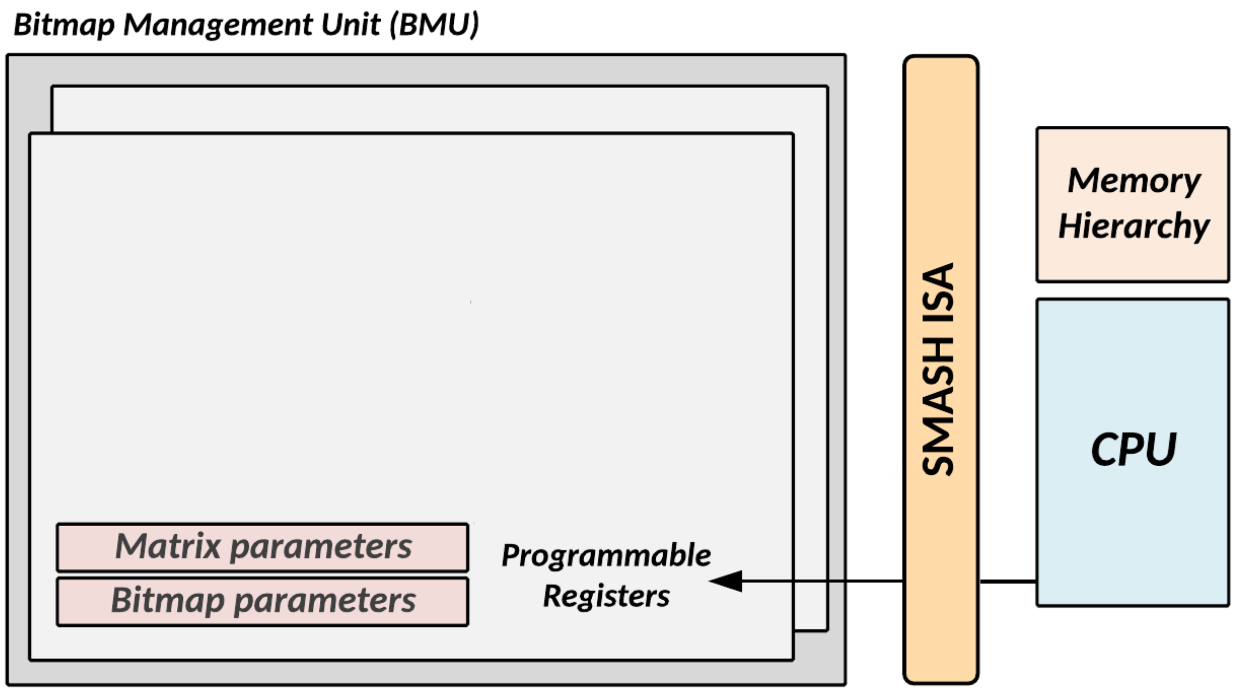
SMASH: Hardware Acceleration Scheme

- Communication over **SMASH ISA**



SMASH: Hardware Acceleration Scheme

- Communication over **SMASH ISA**
- Matrix size and compression ratios as **parameters**



SMASH: Hardware Acceleration Scheme

- Communication over **SMASH ISA**
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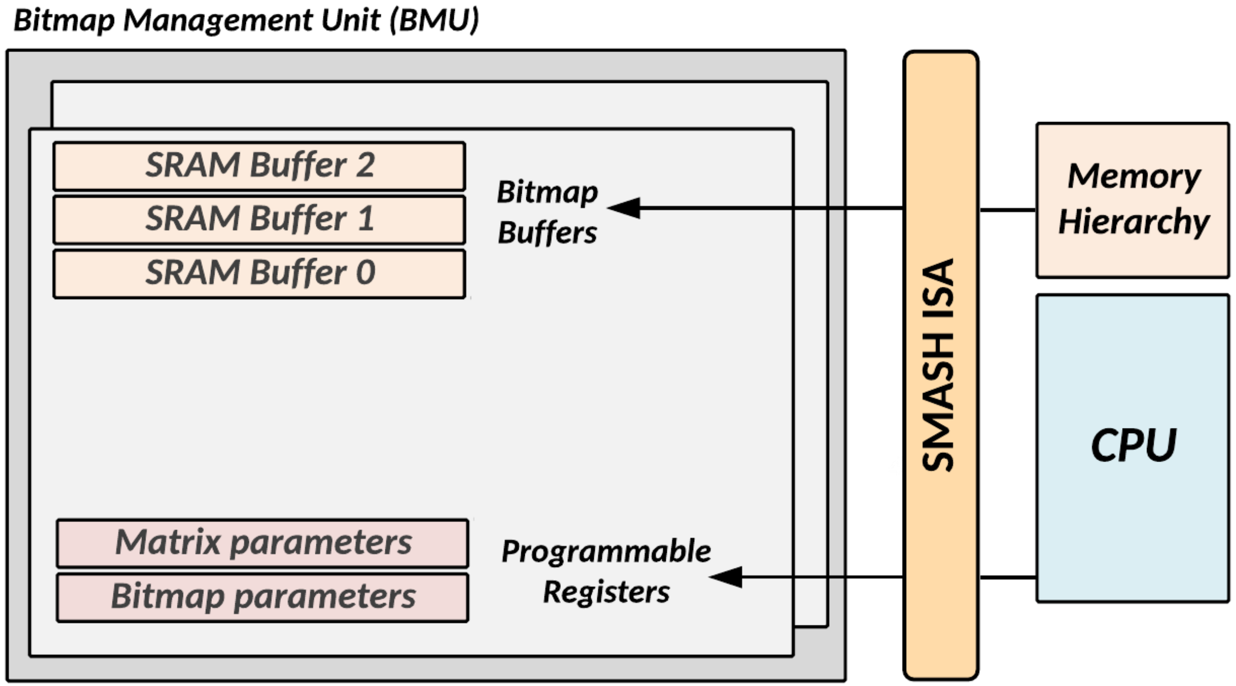
- One **buffer** per bitmap level



SMASH: Hardware Acceleration Scheme

- Communication over **SMASH ISA**
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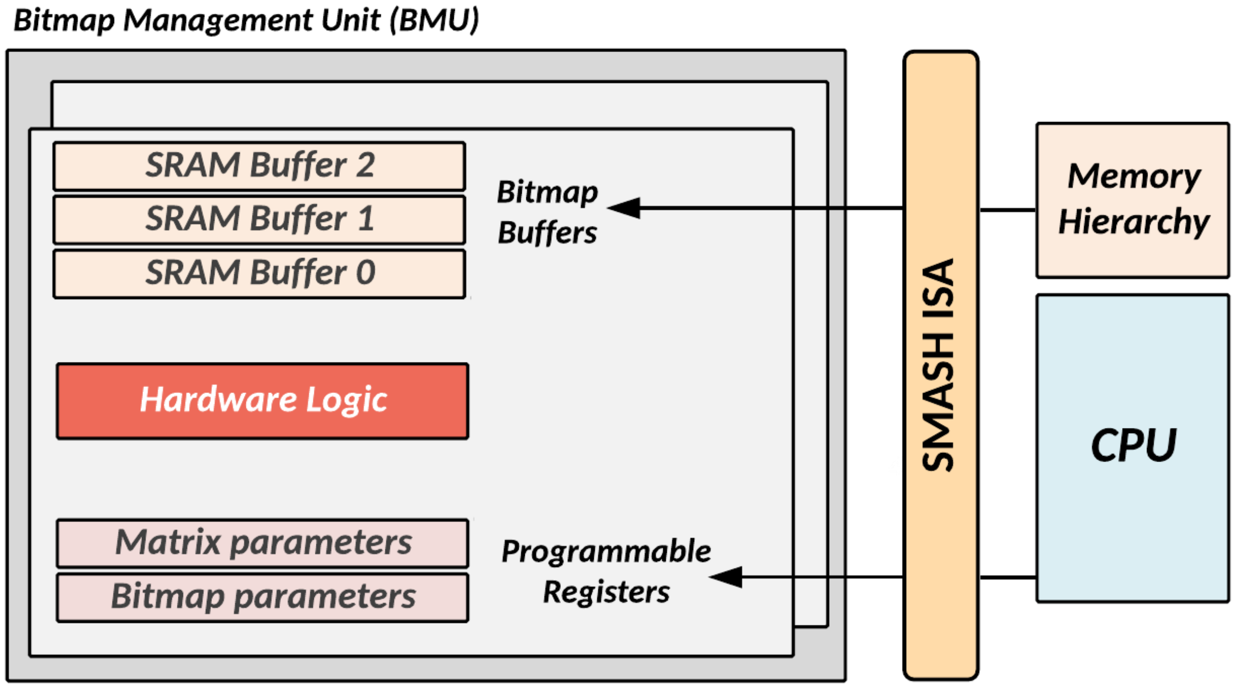
- One **buffer** per bitmap level
- Hardware logic:



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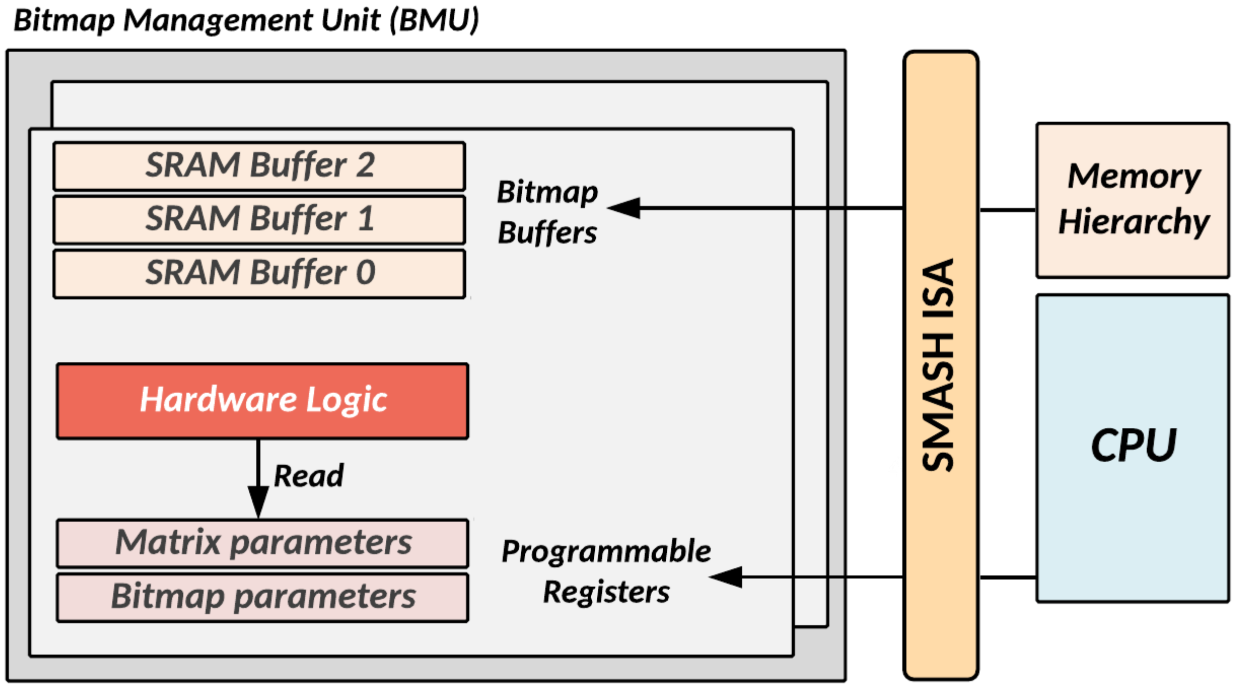
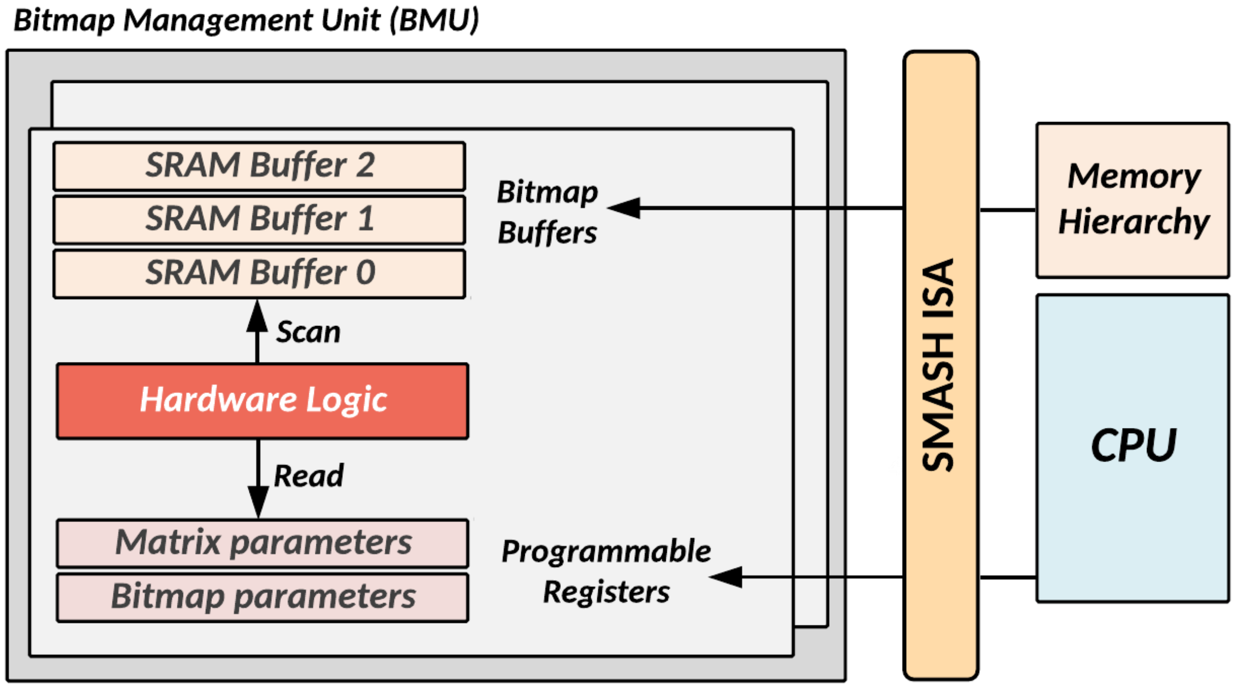


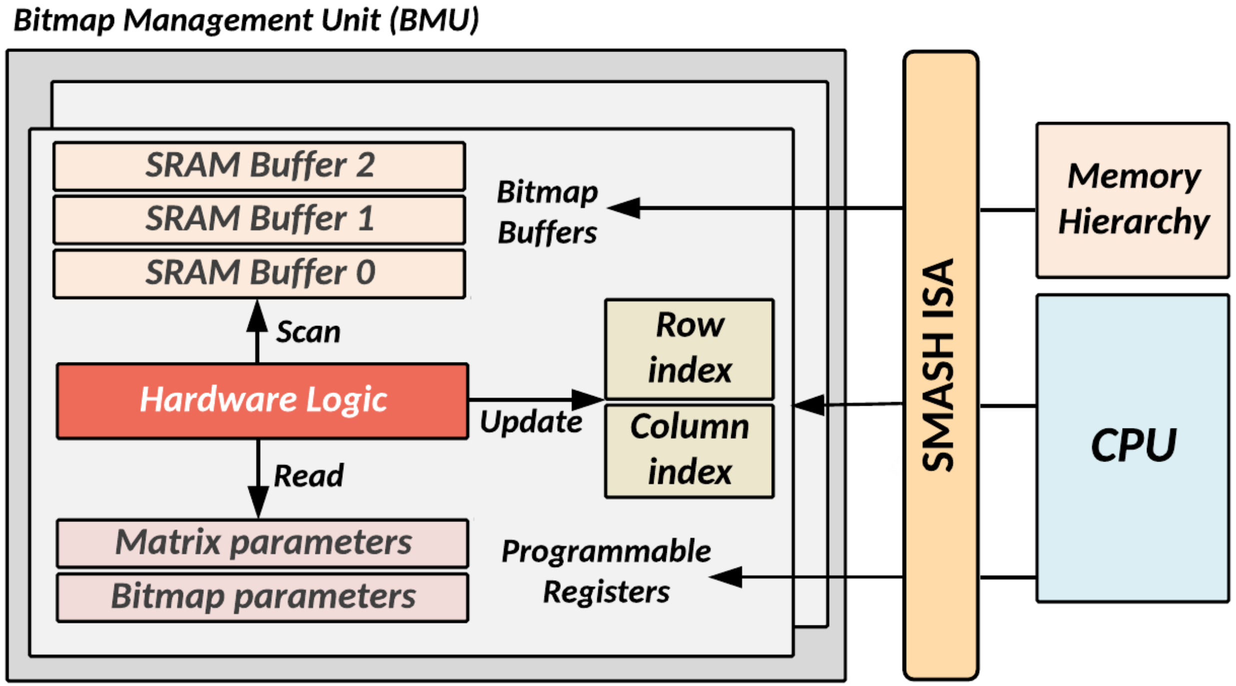
SMASH: Hardware Acceleration Scheme

- Communication over **SMASH ISA**
- Matrix size and compression ratios as **parameters**
- One **buffer** per bitmap level
- Hardware logic:
  - **Reads** parameters
  - **Scans** bitmaps for set bits



SMASH: Hardware Acceleration Scheme

- Communication over **SMASH ISA**
- Matrix size and compression ratios as **parameters**
- One **buffer** per bitmap level
- Hardware logic:
  - **Reads** parameters
  - **Scans** bitmaps for set bits
  - **Computes** index of next **non-zero block** and **stores** it into registers



Evaluation Methodology

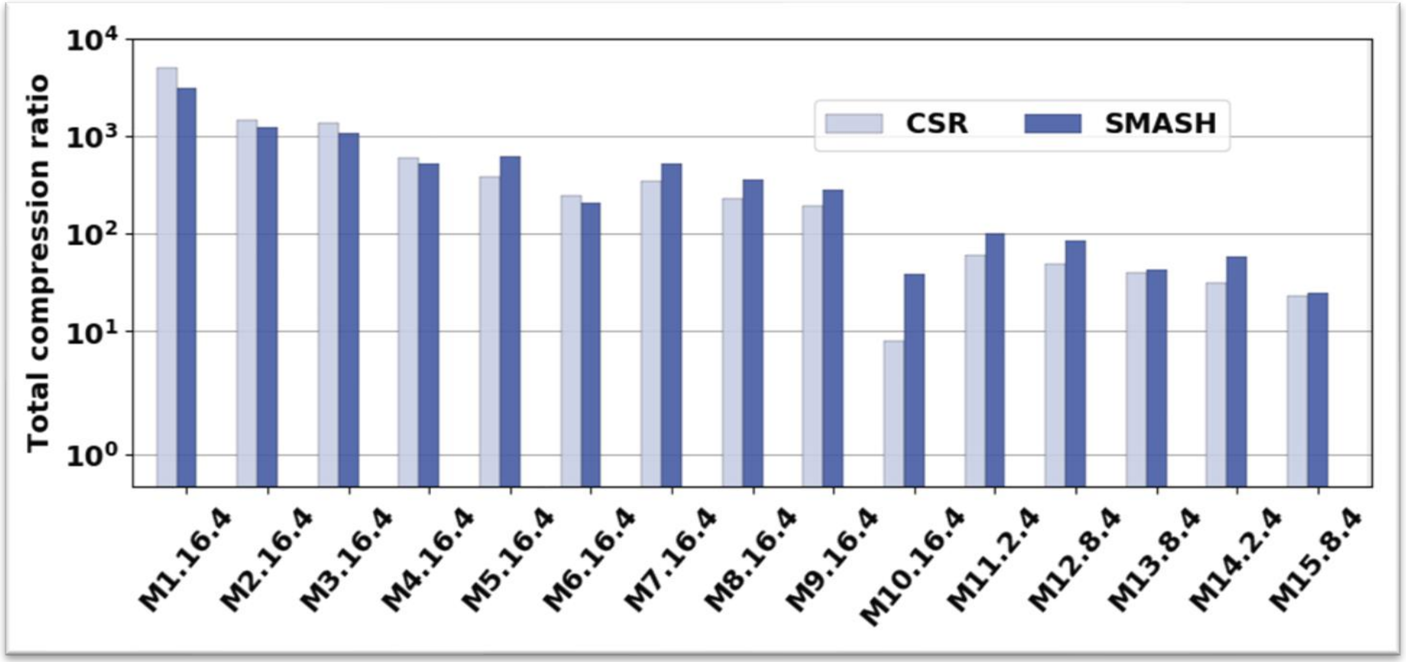
Simulator: ZSim Simulator

Workloads:

- **SpMV & SpMM**
  - 15 matrices
  - **Varied sparsity levels and non-zero distributions**
  - Sparsity level from 0.01% to 8.79%
- **PageRank & Betweenness Centrality**
  - 4 graphs
  - Number of edges from 1M to 3.3M

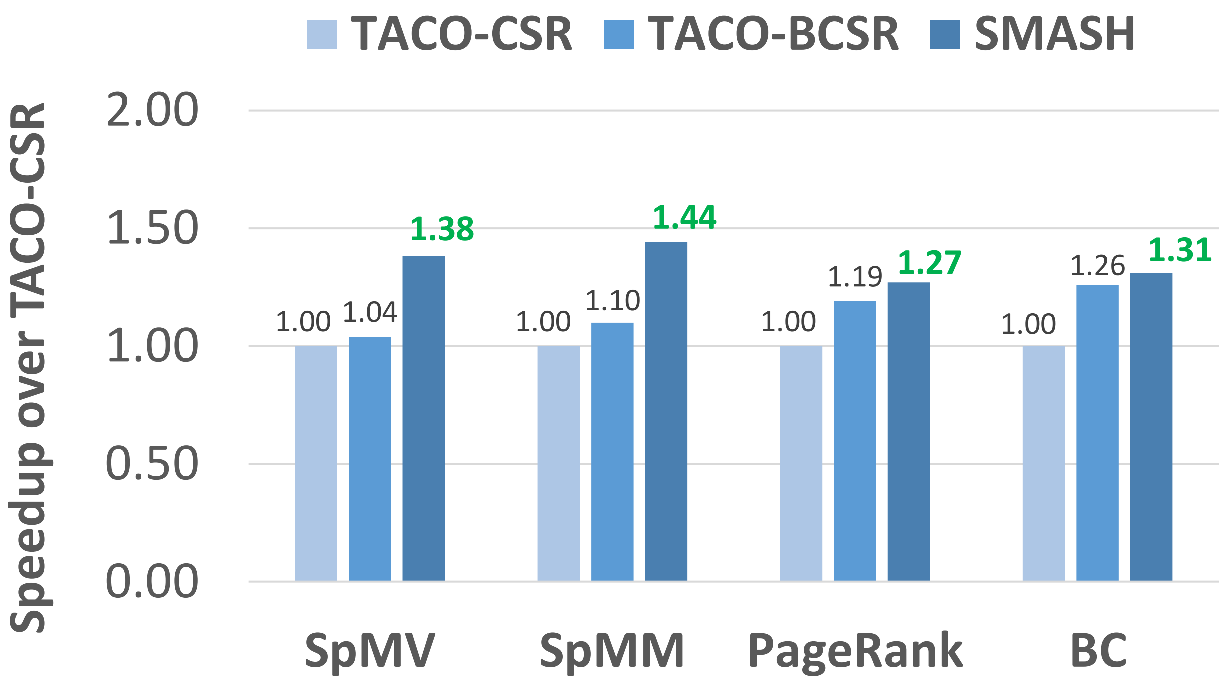
Evaluation Result: Storage Efficiency

1. **Storage** Efficiency
   - **CSR** is better for **very sparse** matrices
   - **SMASH** is better for **denser** matrices
   - Cost of storing **zeros** vs. Cost of storing **indices**



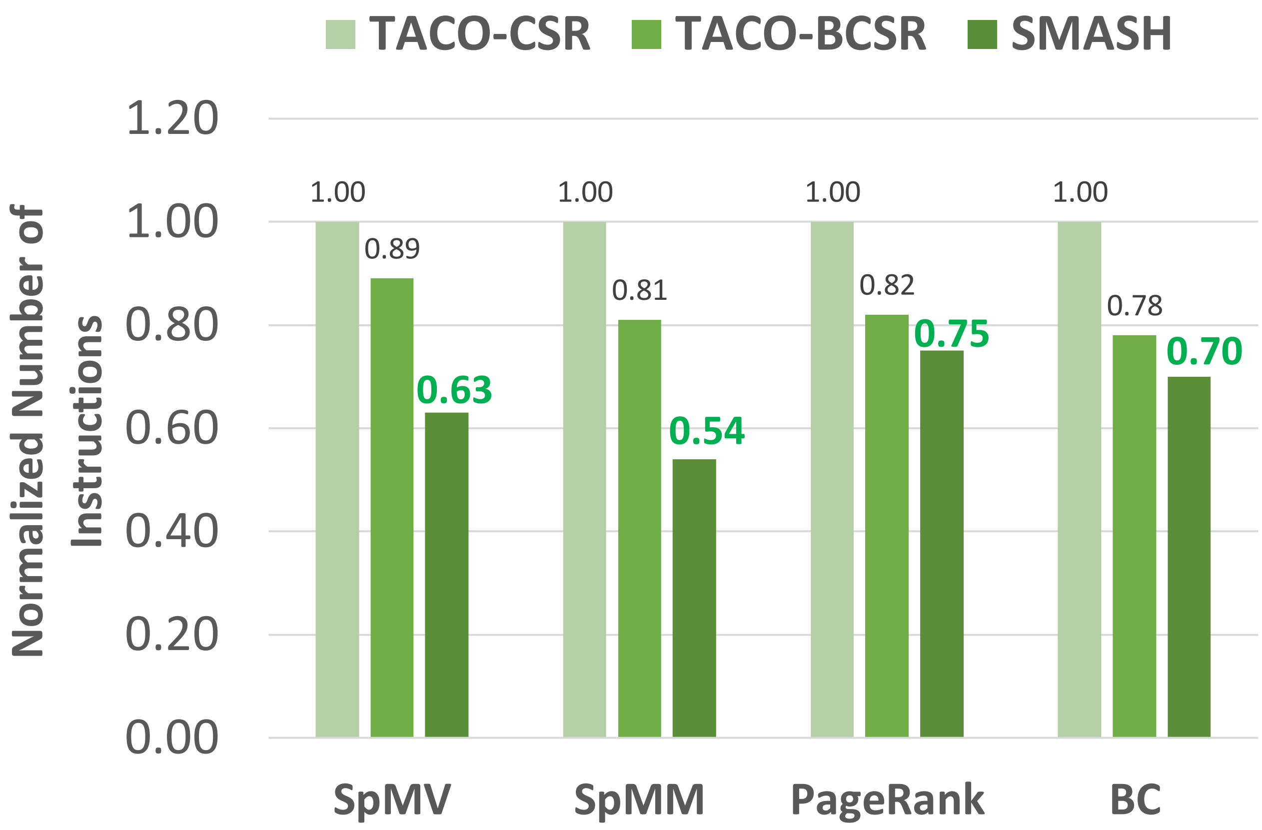
Evaluation Result: Computational Efficiency

1. **Storage** Efficiency
2. **Computational** Efficiency
   - Sparse Matrix Kernels
     - 40 % speedup compared to **CSR**
     - 30% speedup compared to **BCSR**
   - Lower speedup for graph computations



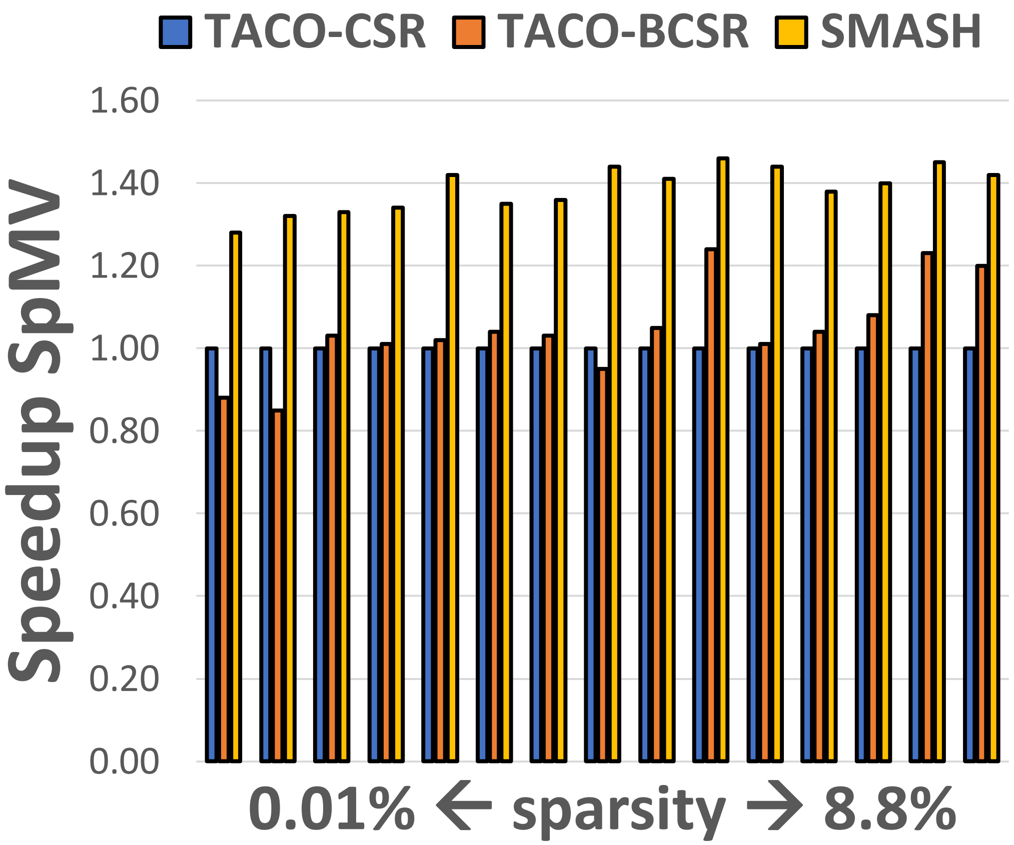
Evaluation Result: Computational Efficiency

1. **Storage** Efficiency
2. **Computational** Efficiency
   - SMASH requires fewer instructions compared to **CSR** and **BCSR**



Evaluation Result: General Applicability

1. **Storage** Efficiency
2. **Computational** Efficiency
3. **General Applicability**
   - Matrices with varied size, structure, and sparsity
   - Increasing the density of a matrix leads to higher indexing overhead

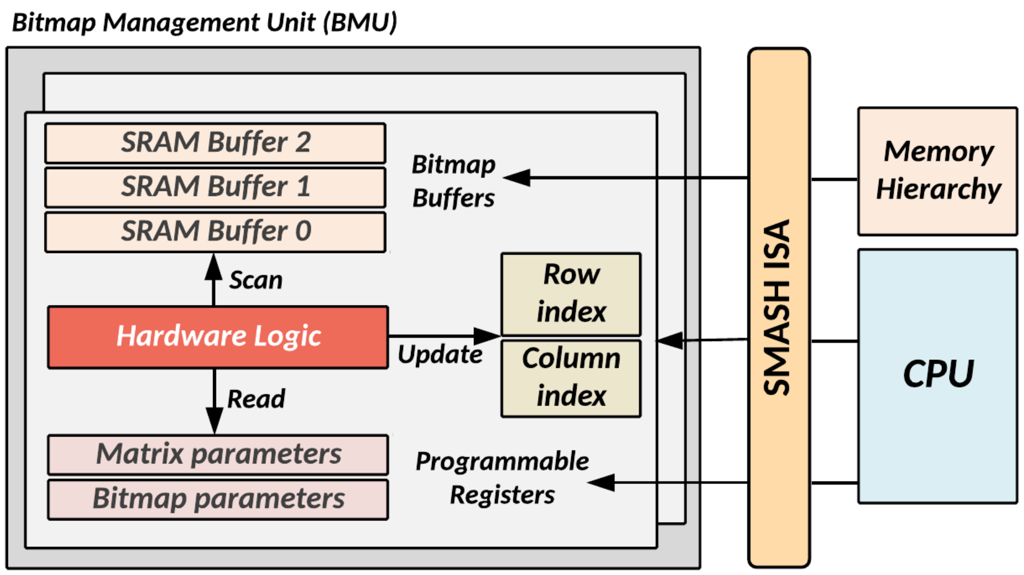


Evaluation Result: Die Area Overhead

BMU

- Support **4 matrices**
- **3 bitmap buffers** per matrix, each with 256 bytes of storage
- 140 bytes for **registers & counters**

0.076% area overhead over an Intel Xeon E5-2698 CPU core



Conclusion

- Many applications heavily rely on **sparse linear algebra**
- They require **effective compression formats** to mitigate **computational and storage overheads**
- Existing **formats** suffer from the **expensive cost** of finding the **position of non-zero elements**
- **SMASH**: **Hardware/Software cooperative mechanism** for **efficient non-zero elements discovery** to accelerate **sparse matrix operations**
  - **Software**: Efficient compression with a **hierarchy of bitmaps**
  - **Hardware**: Scans bitmaps to **find indices of non-zero elements**
- **38 - 44 %** faster than the state-of-the-art for **SpMV and SpMM**
- SMASH is **highly effective, low cost, and widely applicable**

Strengths

- Identifies **indexing as a key bottleneck** in sparse matrix compression schemes
- Among the first to propose a **hardware/software cooperative scheme** to accelerate sparse matrix indexing
- **General applicable**
  - CPU, GPU, other hardware accelerators etc.
  - Any sparse operation like Sparse LU, Sparse QR, etc.
- **Extensive evaluation** from many perspectives
  - Impact of compression ratio, sparsity levels, non-zero distribution, conversion overhead, software-only approach, die area overhead, etc.
- SMASH's implementation of the matrix kernels is **open source**
- **Well written**
  - Little **prior knowledge** required
  - Structure feels **very familiar…**

Weaknesses

- Finding of **optimal compression ratios** might be complicated
- Selection of **number of bitmap levels** is not explained
- Evaluation of **hardware-accelerated SMASH** only in a **simulator**
- **Dynamic updates** of sparse matrix is expensive
  - Requires reload of bitmaps into BMU to avoid coherency issues
  - Dynamic insertion of a non-zero value might require a large copy operation on the non-zero array

Thoughts, Ideas and Alternatives

- SMASH stores zero-containing **blocks** like BCSR, but only as **1D arrays**
  - Partitioning of matrix into **2D regions** would enable **blocking optimization**

0	0
1	0
0	0
0	0
1	0
1	0
0	0
0	0

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

0	1	0	0
0	0	0	0
1	1	0	0
0	0	0	0

- **Lossy compression formats** that approximate matrix values to reduce the memory footprint
  - CSR: Merge integer index (4 bytes) and float value (4 bytes) into a single value (4 bytes)
  - Value stores **index in the leftmost bits** and **value in the remaining bits**
  - **Loss of precision** for floating points values
  - **Loss of range** for integer indices

Thoughts, Ideas and Alternatives

SparTen: A Sparse Tensor Accelerator for Convolutional Neural Networks [Gondimalla et al., 2019]

- Acceleration of **sparse dot products** which are heavily used in CNN
- Representation of sparse vectors with **bit masks to indicate position of non-zero values**
- **Fast computation of the intersection** between non-zero value indices
- Authors note that **sparsity in machine learning models is lower than in other applications** (10% sparsity and more).
- They provide an analysis on when bitmasks are more efficient than conventional formats like CSR.

Thoughts, Ideas and Alternatives

ExTensor: An Accelerator for Sparse Tensor Algebra
[Hegde et al., 2019]

- Acceleration of **sparse linear algebra**
- Representation of sparse operands as **hierarchical trees**
- **Fast computation of the intersection** between non-zero element nodes or non-zero region subtrees to **avoid unnecessary scalar multiplications or even dot products**

Thoughts, Ideas and Alternatives

SIGMA: A Sparse and Irregular GEMM Accelerator with Flexible Interconnects for DNN Training [Qin et al., 2020]

- Acceleration of **sparse matrix-matrix multiplications** in DNN
- Showcase of the **importance** of matrix-matrix multiplications in **deep learning**
- Explanation on why **GPUs and TPUs cannot exploit sparsity** effectively
- As part of their architecture, they use a **bitmap format** to store operands
    - **Constant meta-data overhead** by using one bit per element
    - For denser matrices, bitmaps have **lower overhead** compared to **conventional formats**

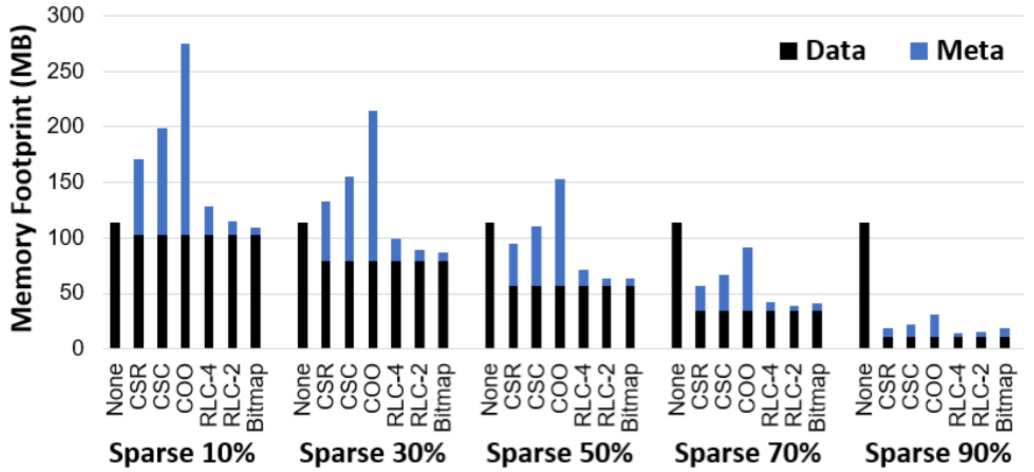


Figure 7: Matrix memory overhead with dimensions M=1632 and K=36548. Format comparisons include: None, CSR, CSC, COO, RLC-4, RLC-2, and Bitmap in the following order.

Key Takeaways

- It is essential to take the **sparsity of matrix operands** into account, if one wants to enable **applications** that operate on **very large and sparse matrices**
- While it takes more time to detect the **root of a problem**, resolving a problem at that level **can benefit a lot of** applications
    - Today's root: Sparse matrix indexing
- One can **benefit from orthogonal concepts of other approaches**
    - BCSR's blocking optimization has the potential to improve SMASH even further

Questions?

Discussion

- How does the **distribution of non-zero elements** affect SMASH?
- How can we reduce the **overhead of zero element computations**?
- How can we use SMASH to accelerate **parallelized matrix computations**?
- Where should we employ **approximation** to reduce the indexing overhead?
  - Directly in the compression format?
  - Approximate memory?
  - Other methods?
- Which **concepts from previous presentations** can be used to reduce the indexing overhead?
- Can you think of any other applications that can benefit from the **use of hierarchical bitmaps**?

Reducing Overhead of Zero Element Computations

Page Overlays:
An Enhanced Virtual Memory Framework to Enable Fine-grained Memory Management
[Seshadri et al., 2015]

- Can significantly boost **system performance and efficiency** while largely retaining the structure of the **existing virtual memory framework**
- Enables the mapping of virtual pages to **overlays** which only **contain a few cache lines**
- With overlays the fetching of and the computation with **zero-only cache lines of SMASH's non-zero blocks** can be avoided
- Can be used to enable **more efficient sparse matrix updates**

Reducing Overhead with Approximate Memory

EDEN:
Enabling Energy-Efficient, High-Performance Deep Neural Network Inference Using Approximate DRAM
[Koppula et al., 2019]

- Nina Richter will present this paper next week, so I will not spoil her presentation today ☺