

Digital Design & Computer Arch.

Lecture 7: Hardware Description Languages and Verilog

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Required Readings (This Week)

- Hardware Description Languages and Verilog
 - H&H Chapter 4 in full
- Timing and Verification
 - H&H Chapters 2.9 and 3.5 + (start Chapter 5)
- By tomorrow, make sure you are done with
 - **P&P Chapters 1-3 + H&H Chapters 1-4**

Required Readings (Next Week)

■ Von Neumann Model, LC-3, and MIPS

- ❑ P&P, Chapter 4, 5
- ❑ H&H, Chapter 6
- ❑ P&P, Appendices A and C (ISA and microarchitecture of LC-3)
- ❑ H&H, Appendix B (MIPS instructions)

■ Programming

- ❑ P&P, Chapter 6

■ **Recommended:** Digital Building Blocks

- ❑ H&H, Chapter 5

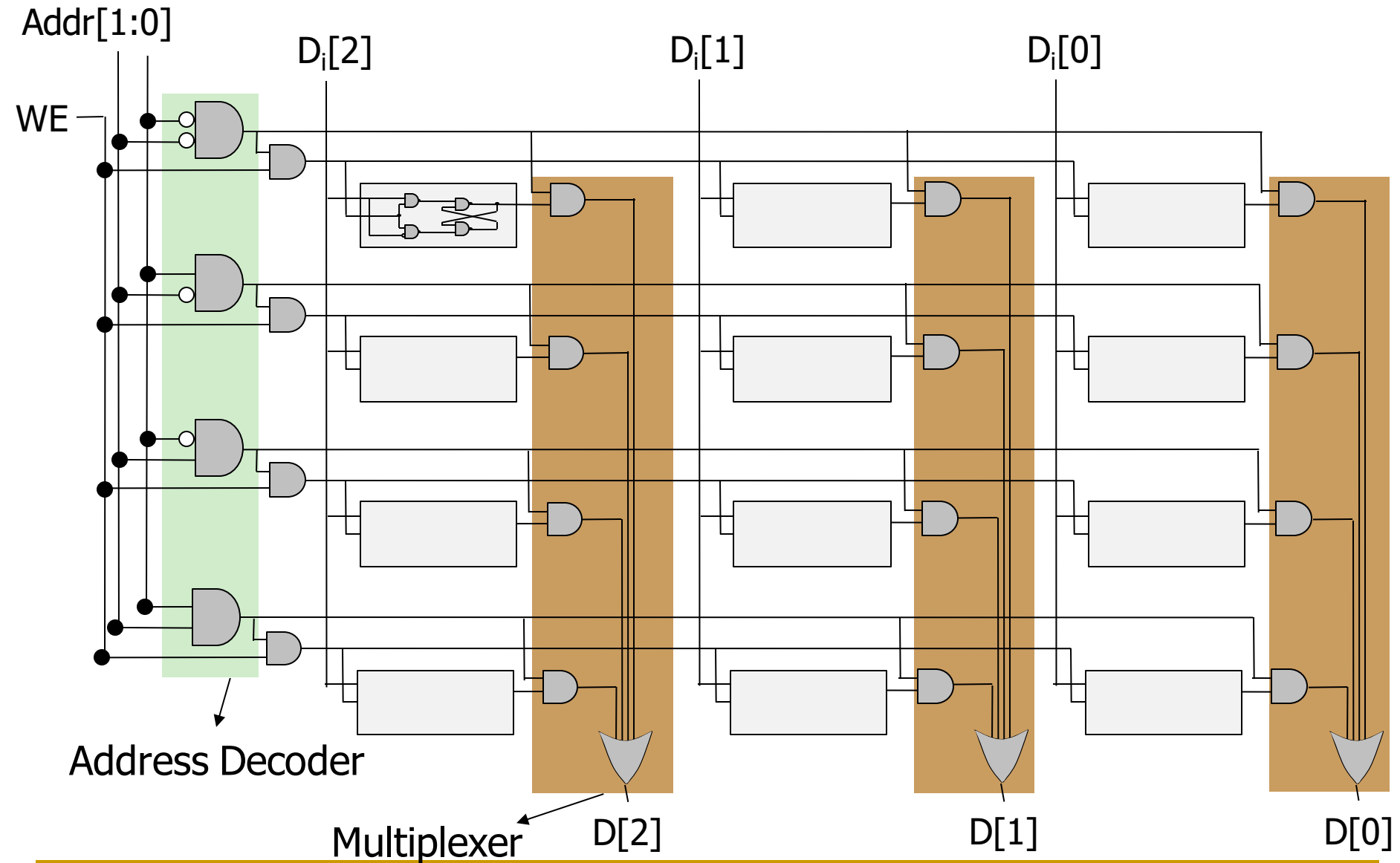
Agenda

- Hardware Description Languages
- Implementing Combinational Logic (in Verilog)
- Implementing Sequential Logic (in Verilog)

- The Verilog slides constitute a tutorial. We will not cover all.
- All slides will be beneficial for your labs.

Aside: Implementing Logic Functions Using Memory

Recall: A Bigger Memory Array (4 locations X 3 bits)



Memory-Based Lookup Table Example

- Memory arrays can also perform Boolean Logic functions
 - 2^N -location M -bit memory can perform any N -input, M -output function
 - Lookup Table (LUT): Memory array used to perform logic functions
 - Each address: row in truth table; each data bit: corresponding output value

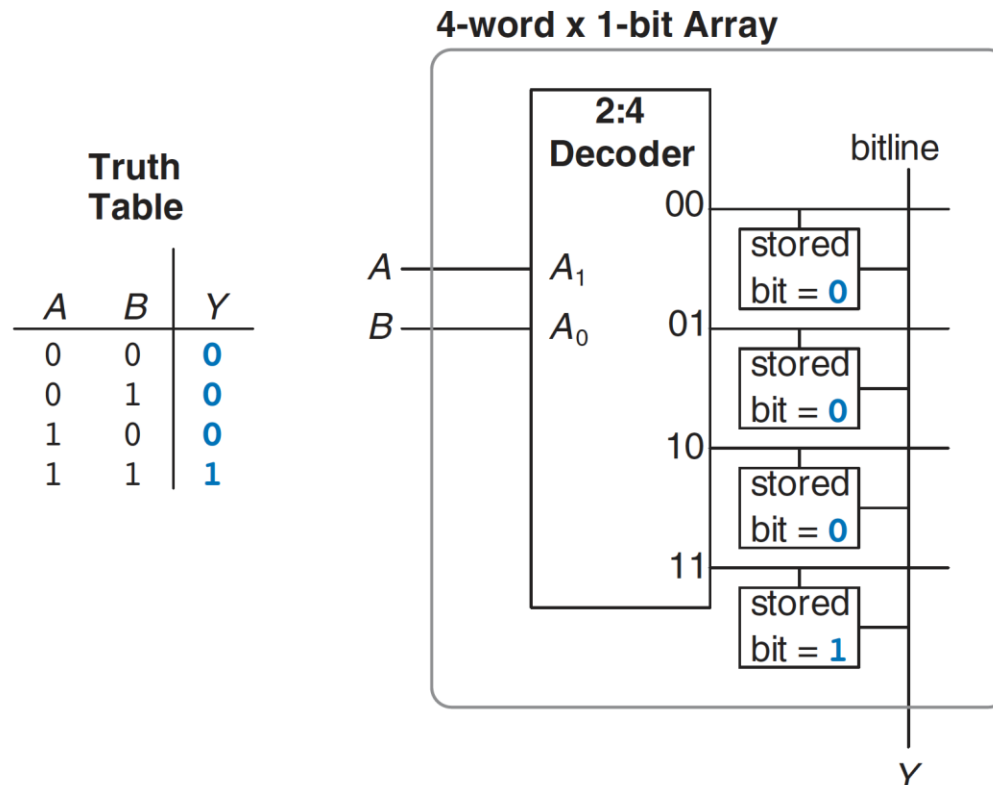
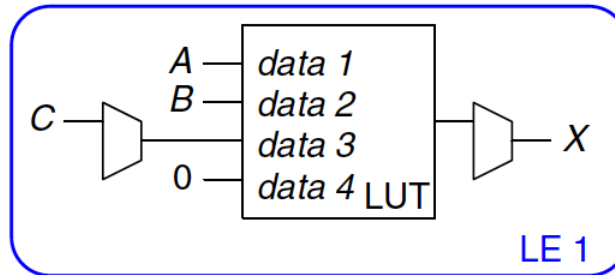


Figure 5.52 4-word \times 1-bit memory array used as a lookup table

Lookup Tables (LUTs)

- LUTs are commonly used in FPGAs
 - To enable programmable/reconfigurable logic functions
 - To enable easy integration of combinational and sequential logic

(A) data 1	(B) data 2	(C) data 3	data 4	(X) LUT output
0	0	0	X	0
0	0	1	X	1
0	1	0	X	0
0	1	1	X	0
1	0	0	X	0
1	0	1	X	0
1	1	0	X	1
1	1	1	X	0



(A) data 1	(B) data 2	data 3	data 4	(Y) LUT output
0	0	X	X	0
0	1	X	X	0
1	0	X	X	1
1	1	X	X	0

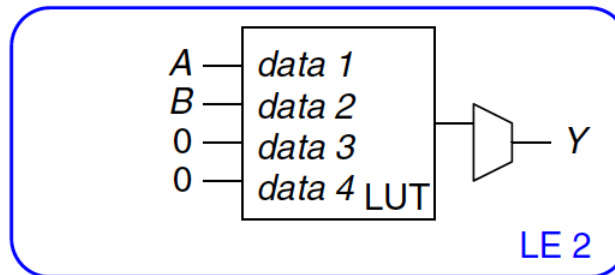
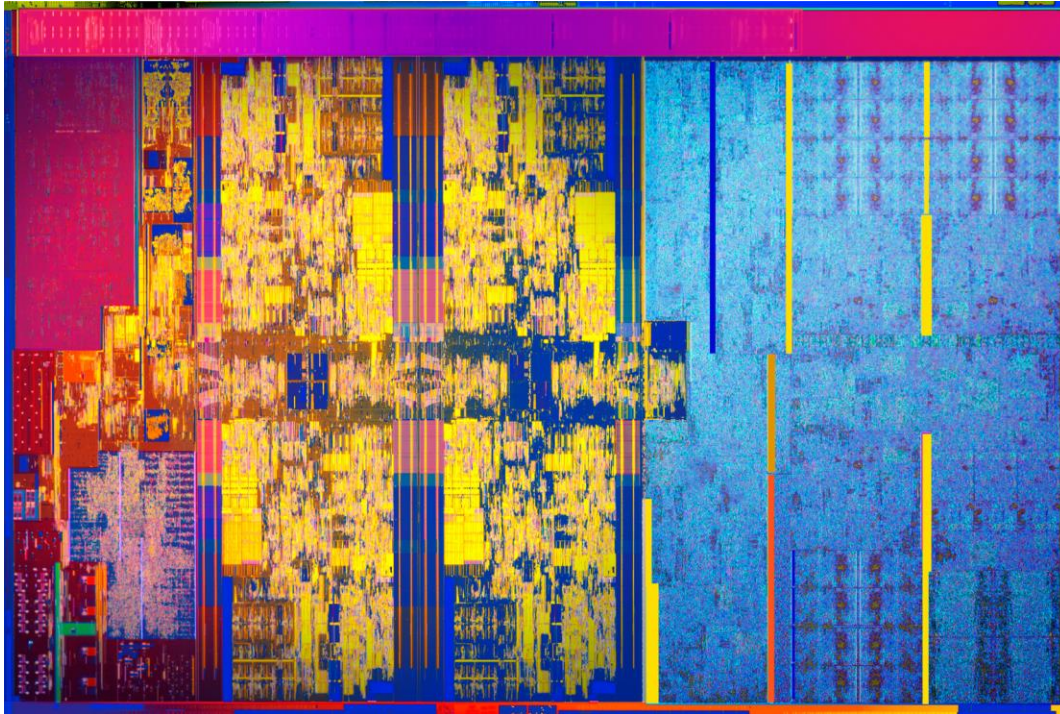


Figure 5.59 LE configuration for two functions of up to four inputs each

Hardware Description Languages & Verilog

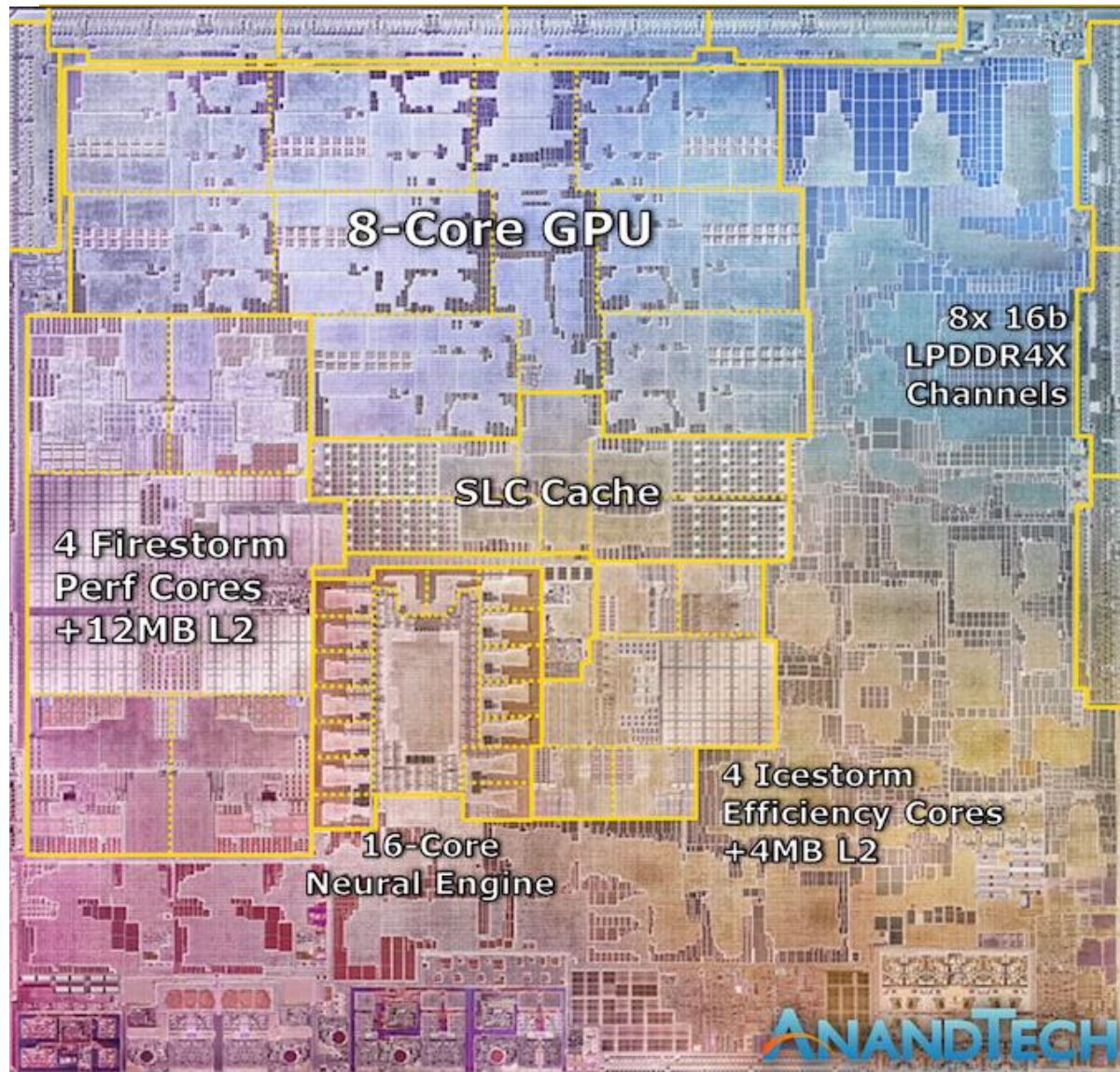
2017: Intel Kaby Lake



https://en.wikichip.org/wiki/intel/microarchitectures/kaby_lake

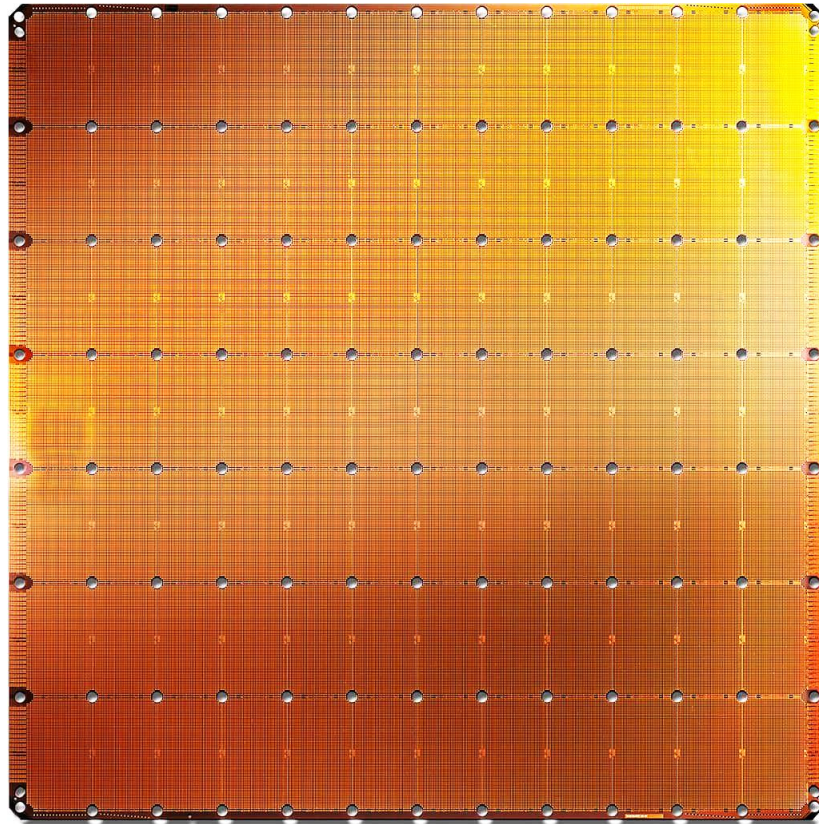
- 64-bit processor
- 4 cores, 8 threads
- 14-19 stage pipeline
- 3.9 GHz clock freq.
- **1.75B transistors**
- In ~47 years, about 1,000,000-fold growth in transistor count and performance!

2021: Apple M1



- 4 High-Perf GP Cores
- 4 Efficient GP Cores
- 8-Core GPU
- 16-Core Neural Engine
- Lots of Cache
- Many Caches
- 8x Memory Channels
- 16B transistors

2019: Cerebras Wafer Scale Engine



Cerebras WSE
1.2 Trillion transistors
46,225 mm²

- The largest ML accelerator chip
- 400,000 cores



Largest GPU
21.1 Billion transistors
815 mm²

NVIDIA TITAN V

<https://www.anandtech.com/show/14758/hot-chips-31-live-blogs-cerebras-wafer-scale-deep-learning>

<https://www.cerebras.net/cerebras-wafer-scale-engine-why-we-need-big-chips-for-deep-learning/>

How to Deal with This Complexity?

- Hardware Description Languages!
- Needs and wants:
 - Ability to specify complex designs
 - ... and to simulate their behavior (functional & timing)
 - ... and to synthesize (automatically design) portions of it
 - have an error-free path to implementation
- Hardware Description Languages enable all of the above
 - Languages designed to describe and specify hardware
 - There are similarly-featured HDLs (e.g., **Verilog**, VHDL, ...)
 - if you learn one, it is not hard to learn another
 - mapping between languages is typically mechanical, especially for the commonly used subset

Hardware Description Languages

- **Two well-known hardware description languages**

- **Verilog**

- Developed in 1984 by Gateway Design Automation
- Became an IEEE standard (1364) in 1995
- More popular in US

- **VHDL (VHSIC Hardware Description Language)**

- Developed in 1981 by the US Department of Defense
- Became an IEEE standard (1076) in 1987
- More popular in Europe

- We will use Verilog in this course

Hardware Design Using HDL

Principle: Hierarchical Design

■ Design a hierarchy of modules

- ❑ Predefined “primitive” gates (AND, OR, ...)
- ❑ Simple modules are built by instantiating these gates (components like MUXes)
- ❑ Complex modules are built by instantiating simple modules, ...

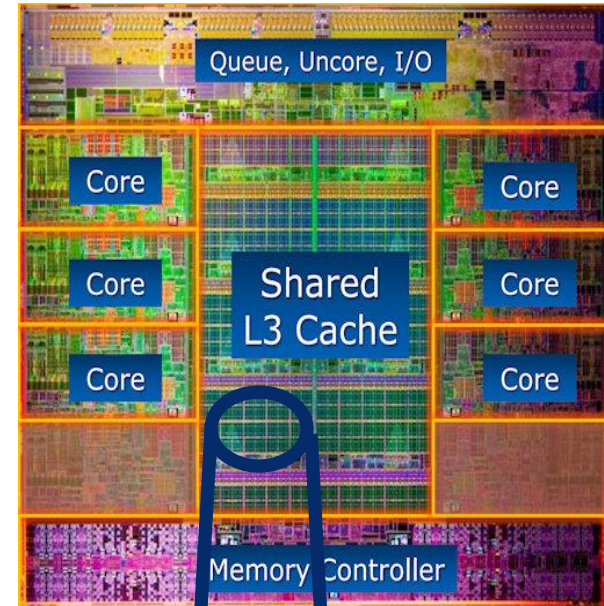
■ Hierarchy controls complexity

- ❑ Analogous to the use of function/method abstraction in programming

■ Complexity is a BIG deal

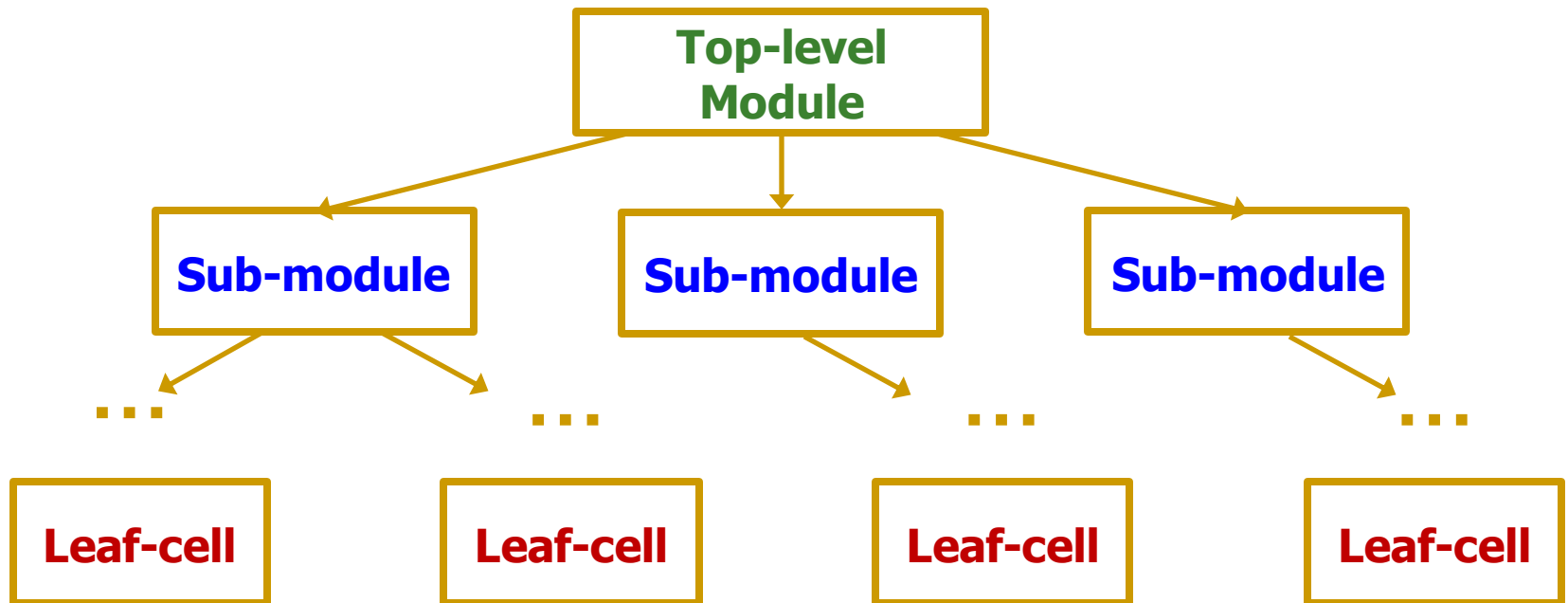
- ❑ In real world, how big is the size of a module (that is described in HDL and then synthesized to gates)?

<https://techreport.com/review/21987/intel-core-i7-3960x-processor>



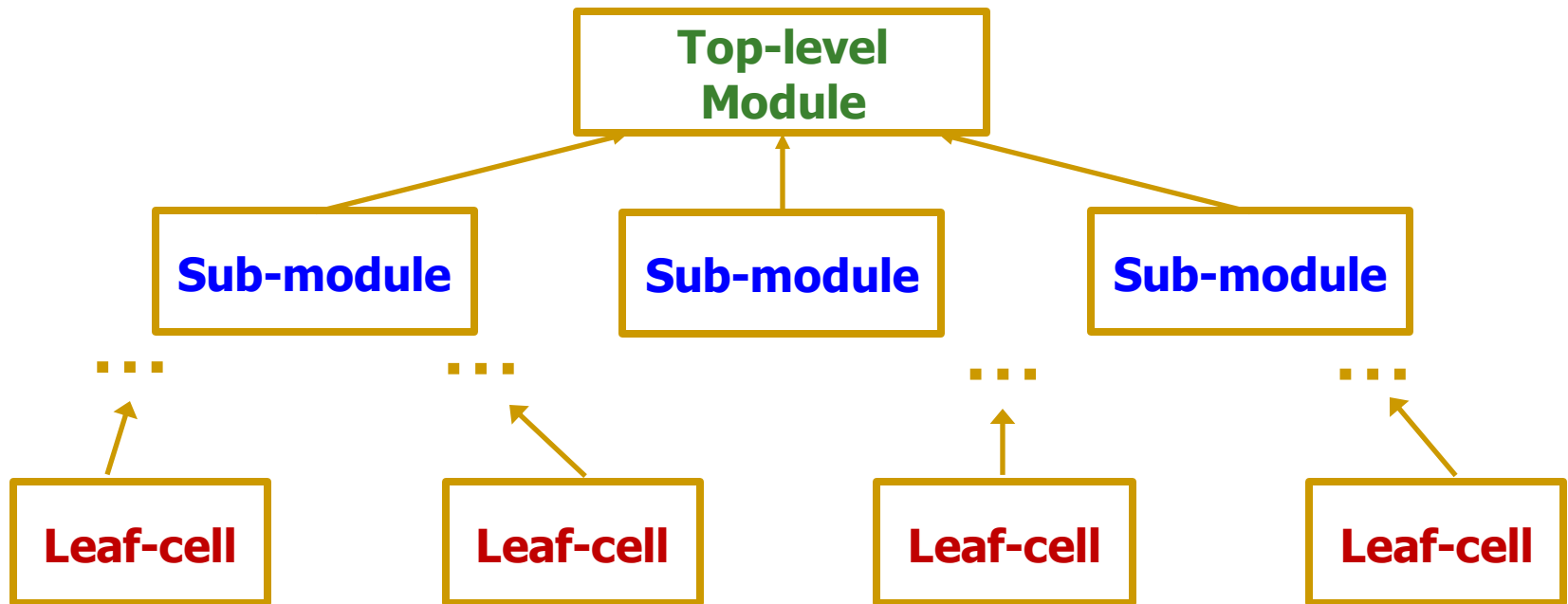
Top-Down Design Methodology

- We define the **top-level module** and identify the **sub-modules** necessary to build the top-level module
- Subdivide the sub-modules until we come to **leaf cells**
 - **Leaf cell**: circuit components that cannot further be divided (e.g., *logic gates, cell libraries*)



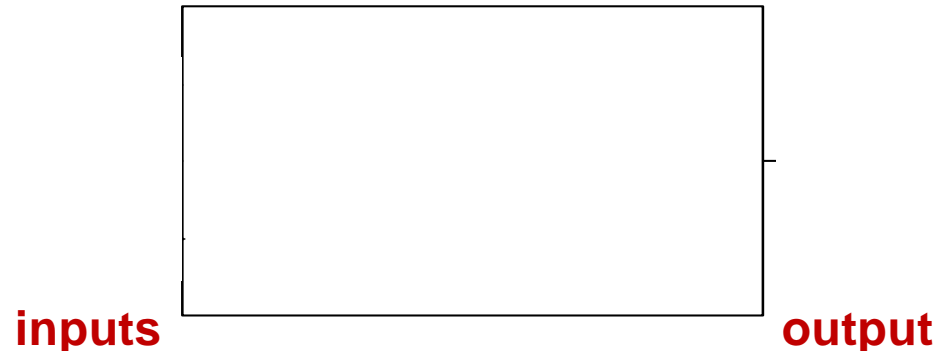
Bottom-Up Design Methodology

- We first identify the **building blocks** that are available to us
- **Build bigger modules**, using these building blocks
- These modules are then used for higher-level modules until we build the **top-level module** in the design

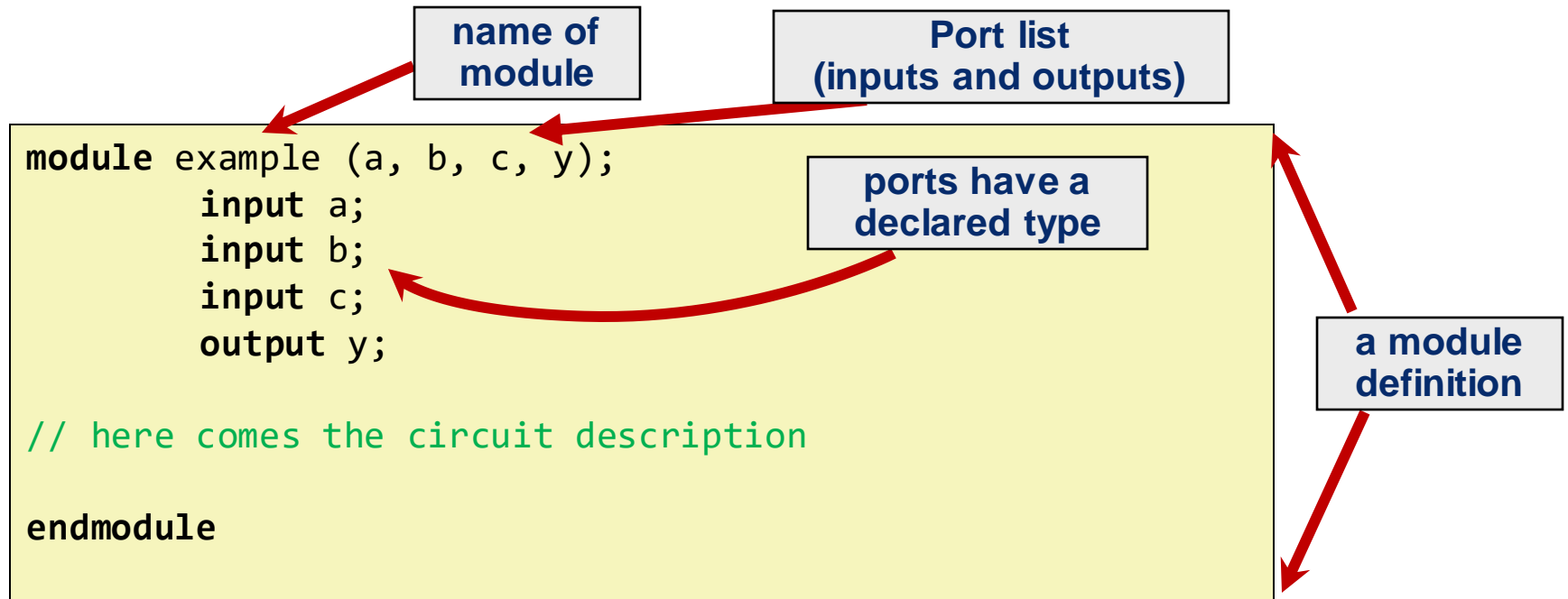
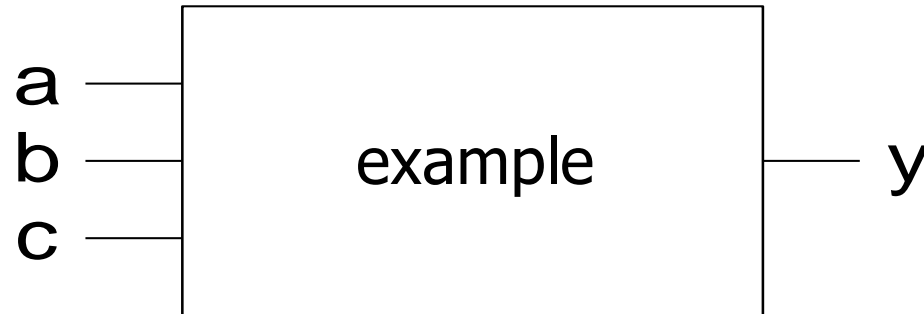


Defining a Module in Verilog

- A **module** is the main building block in Verilog
- We first need to define:
 - **Name** of the module
 - **Directions** of its **ports** (e.g., **input**, **output**)
 - **Names** of its **ports**
- Then:
 - Describe the **functionality** of the module



Implementing a Module in Verilog



A Question of Style

- **The following two codes are functionally identical**

```
module test ( a, b, y );  
    input a;  
    input b;  
    output y;  
  
endmodule
```

```
module test ( input a,  
              input b,  
              output y );  
  
endmodule
```

port name and direction declaration
can be combined

What If We Have Multi-bit Input/Output?

- **You can also define multi-bit Input/Output (Bus)**

- [range_end : range_start]
- **Number of bits:** range_end – range_start + 1

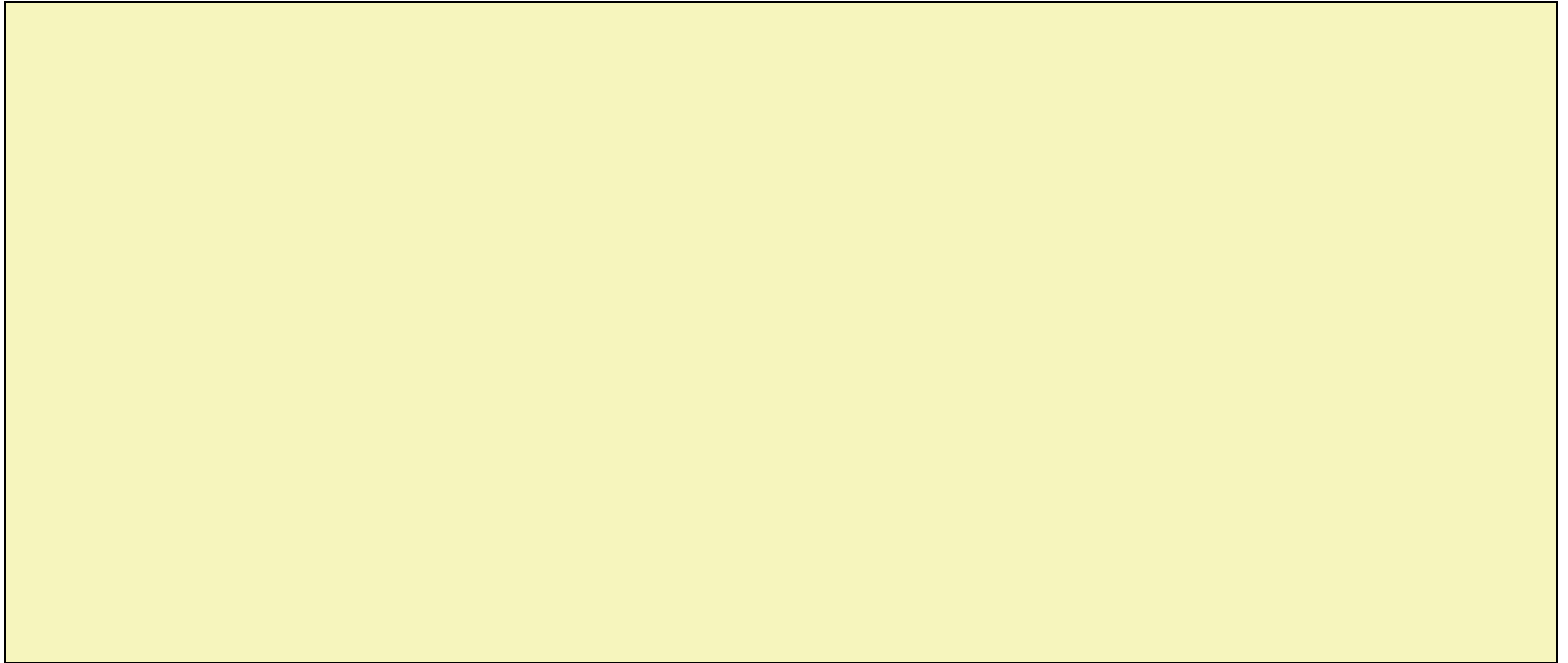
- **Example:**

```
input  [31:0] a;    // a[31], a[30] .. a[0]
output [15:8] b1;   // b1[15], b1[14] .. b1[8]
output [7:0]  b2;   // b2[7], b2[6] .. b2[0]
input                c; // single signal
```

- **a** represents a 32-bit value, so we prefer to define it as:
[31:0] a
- It is preferred over [0:31] a which resembles *array* definition
- It is good practice to **be consistent** with the representation of multi-bit signals, i.e., always [31:0] or always [0:31]

Manipulating Bits

- Bit Slicing
- Concatenation
- Duplication



Basic Syntax

- Verilog is case sensitive
 - `SomeName` and `somename` are not the same!
- Names cannot start with numbers:
 - `2good` is not a valid name
- Whitespaces are ignored

```
// Single line comments start with a //  
  
/* Multiline comments  
   are defined like this */
```


Two Main Styles of HDL Implementation

■ **Structural (Gate-Level)**

- ❑ The module body contains **gate-level description** of the circuit
- ❑ Describe how modules are interconnected
- ❑ Each module contains other modules (instances)
- ❑ ... and interconnections between those modules
- ❑ Describes a hierarchy of modules defined as gates

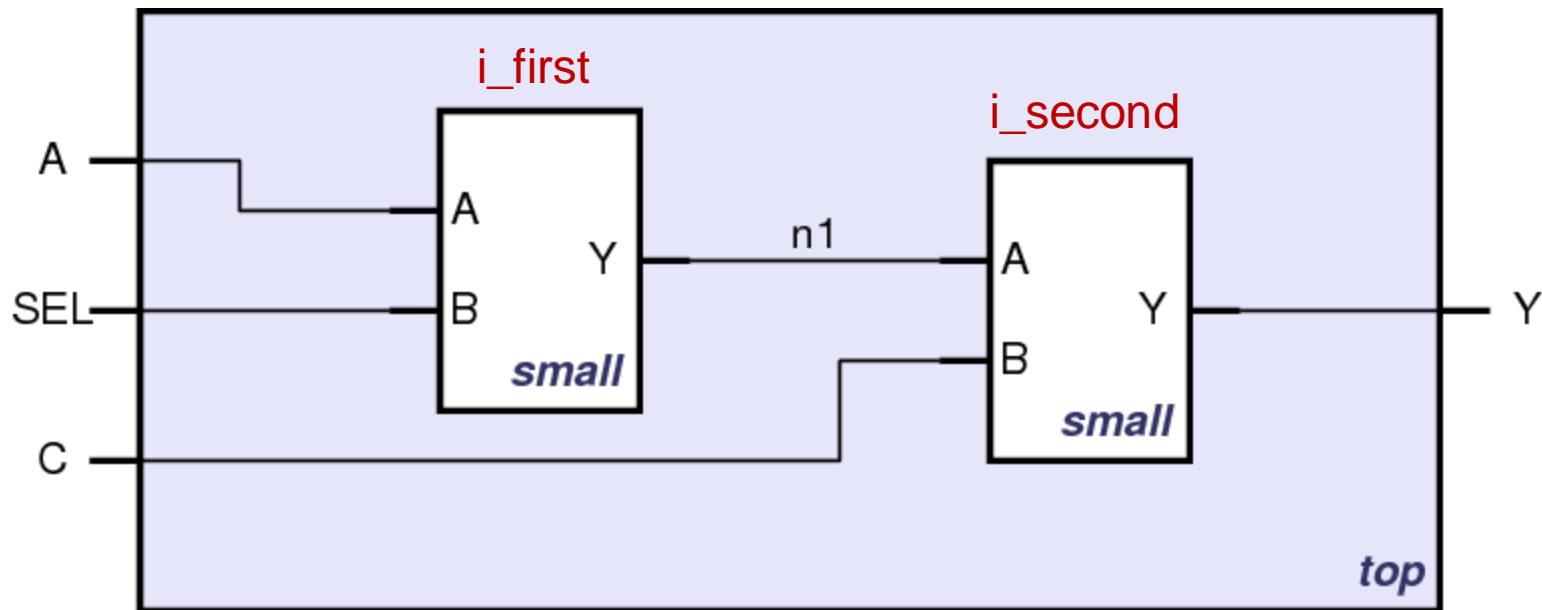
■ **Behavioral**

- ❑ The module body contains **functional description** of the circuit
- ❑ Contains logical and mathematical **operators**
- ❑ **Level of abstraction is higher than gate-level**
 - Many possible gate-level realizations of a behavioral description

■ **Many practical designs use a combination of both**

Structural (Gate-Level) HDL

Structural HDL: Instantiating a Module

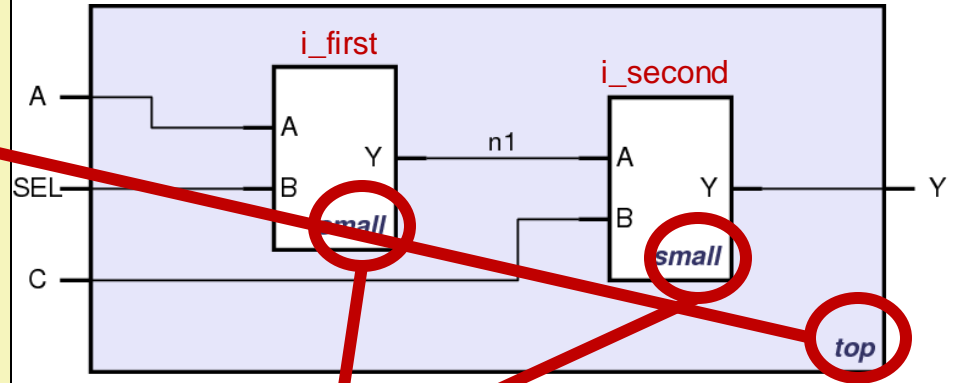


Schematic of module "top" that is built from two instances of module "small"

Structural HDL Example

■ Module Definitions in Verilog

```
module top(A, SEL, C, Y);  
  input A, SEL, C;  
  output Y;  
  wire n1;  
  
endmodule
```

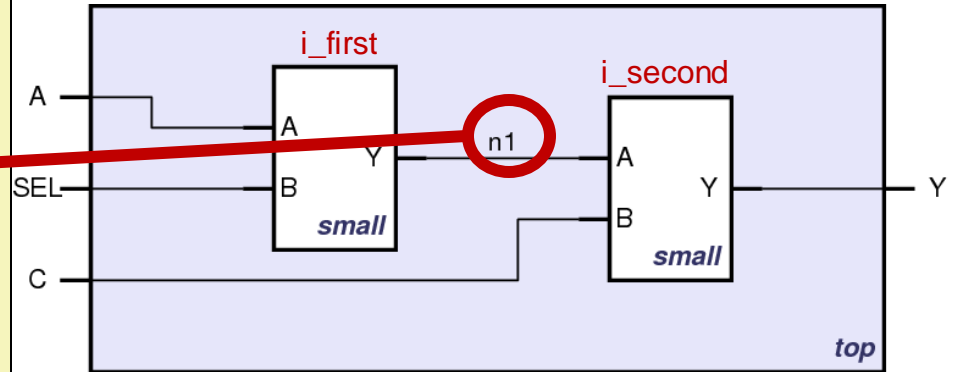


```
module small(A, B, Y);  
  input A;  
  input B;  
  output Y;  
  
  // description of small  
  
endmodule
```

Structural HDL Example

■ Defining wires (module interconnections)

```
module top (A, SEL, C, Y);  
  input A, SEL, C;  
  output Y;  
  wire n1;  
  
endmodule
```



```
module small (A, B, Y);  
  input A;  
  input B;  
  output Y;  
  
  // description of small  
  
endmodule
```

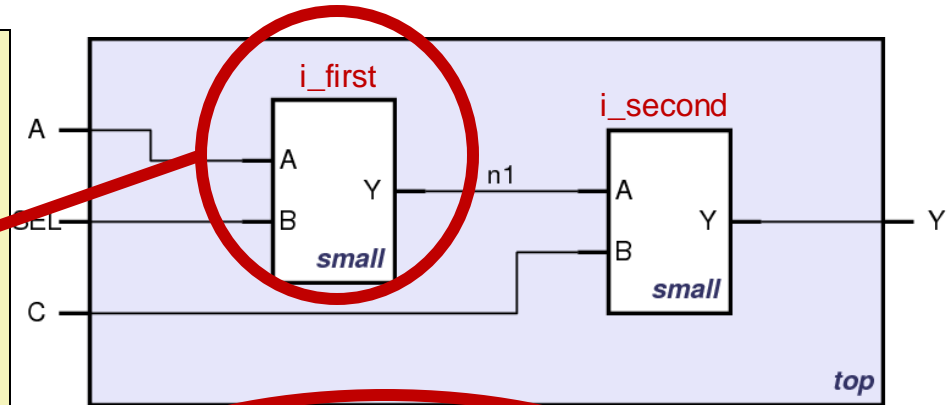
Structural HDL Example

■ The first instantiation of the “small” module

```
module top (A, SEL, C, Y);  
  input A, SEL, C;  
  output Y;  
  wire n1;
```

```
// instantiate small once  
small i_first ( .A(A),  
                .B(SEL),  
                .Y(n1) );
```

```
endmodule
```



```
module small (A, B, Y);  
  input A;  
  input B;  
  output Y;
```

```
// description of small
```

```
endmodule
```

Structural HDL Example

■ The second instantiation of the “small” module

```
module top (A, SEL, C, Y);  
  input A, SEL, C;  
  output Y;  
  wire n1;
```

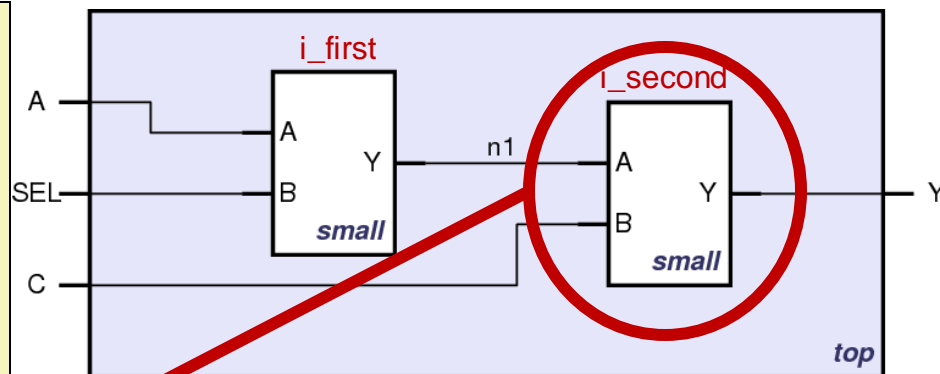
```
// instantiate small once
```

```
small i_first ( .A(A),  
                .B(SEL),  
                .Y(n1) );
```

```
// instantiate small second time
```

```
small i_second ( .A(n1),  
                 .B(C),  
                 .Y(Y) );
```

```
endmodule
```



```
module small (A, B, Y);  
  input A;  
  input B;  
  output Y;
```

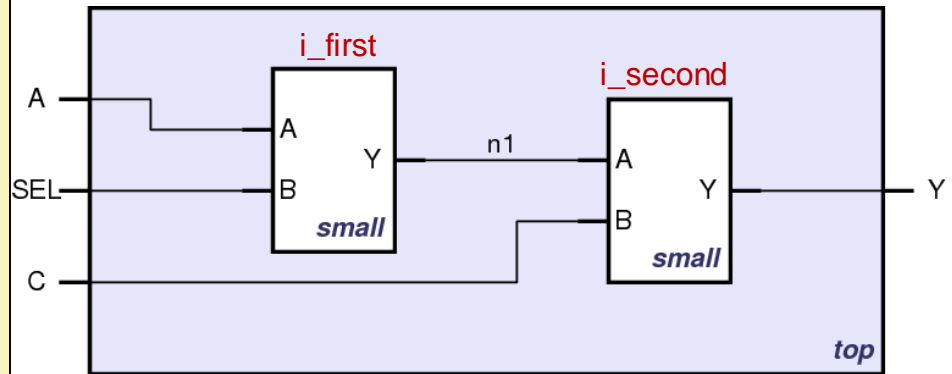
```
// description of small
```

```
endmodule
```

Structural HDL Example

■ Short form of module instantiation

```
module top (A, SEL, C, Y);  
  input A, SEL, C;  
  output Y;  
  wire n1;  
  
  // alternative  
  small i_first ( A, SEL, n1 );  
  
  /* Shorter instantiation,  
     pin order very important */  
  
  // any pin order, safer choice  
  small i_second ( .B(C),  
                  .Y(Y),  
                  .A(n1) );  
  
endmodule
```



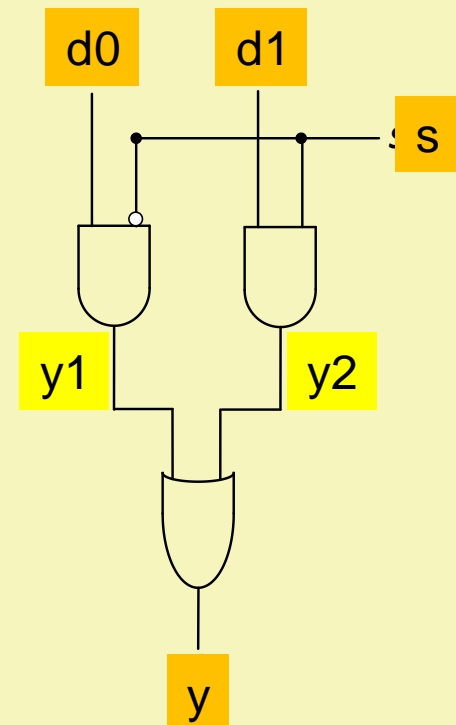
```
module small (A, B, Y);  
  input A;  
  input B;  
  output Y;  
  
  // description of small  
  
endmodule
```

**Short form is not good practice
as it reduces code maintainability**

Structural HDL Example (II)

- Verilog supports basic logic gates as predefined *primitives*
 - These primitives are *instantiated* like modules except that they are predefined in Verilog and *do not need a module definition*

```
module mux2(input d0, d1,  
            input s,  
            output y);  
    wire ns, y1, y2;  
  
    not    g1 (ns, s);  
    and    g2 (y1, d0, ns);  
    and    g3 (y2, d1, s);  
    or     g4 (y, y1, y2);  
  
endmodule
```



Behavioral HDL

Recall: Two Main Styles of HDL Implementation

■ **Structural (Gate-Level)**

- ❑ The module body contains **gate-level description** of the circuit
- ❑ Describe how modules are interconnected
- ❑ Each module contains other modules (instances)
- ❑ ... and interconnections between those modules
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■ **Behavioral**

- ❑ The module body contains **functional description** of the circuit
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 - Many possible gate-level realizations of a behavioral description

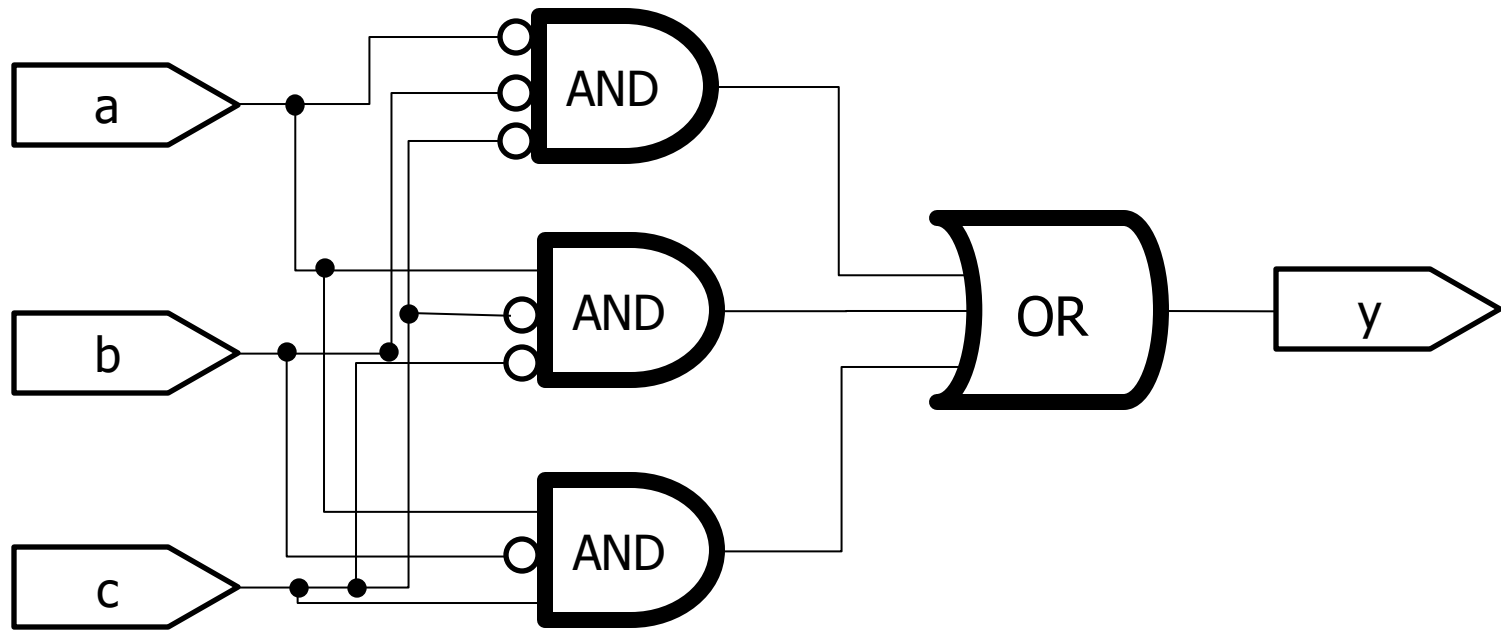
■ **Many practical designs use a combination of both**

Behavioral HDL: Defining Functionality

```
module example (a, b, c, y);  
    input a;  
    input b;  
    input c;  
    output y;  
  
    // here comes the circuit description  
    assign y = ~a & ~b & ~c |  
               a & ~b & ~c |  
               a & ~b & c;  
  
endmodule
```

Behavioral HDL: Schematic View

A behavioral implementation still models a hardware circuit!



Bitwise Operators in Behavioral Verilog

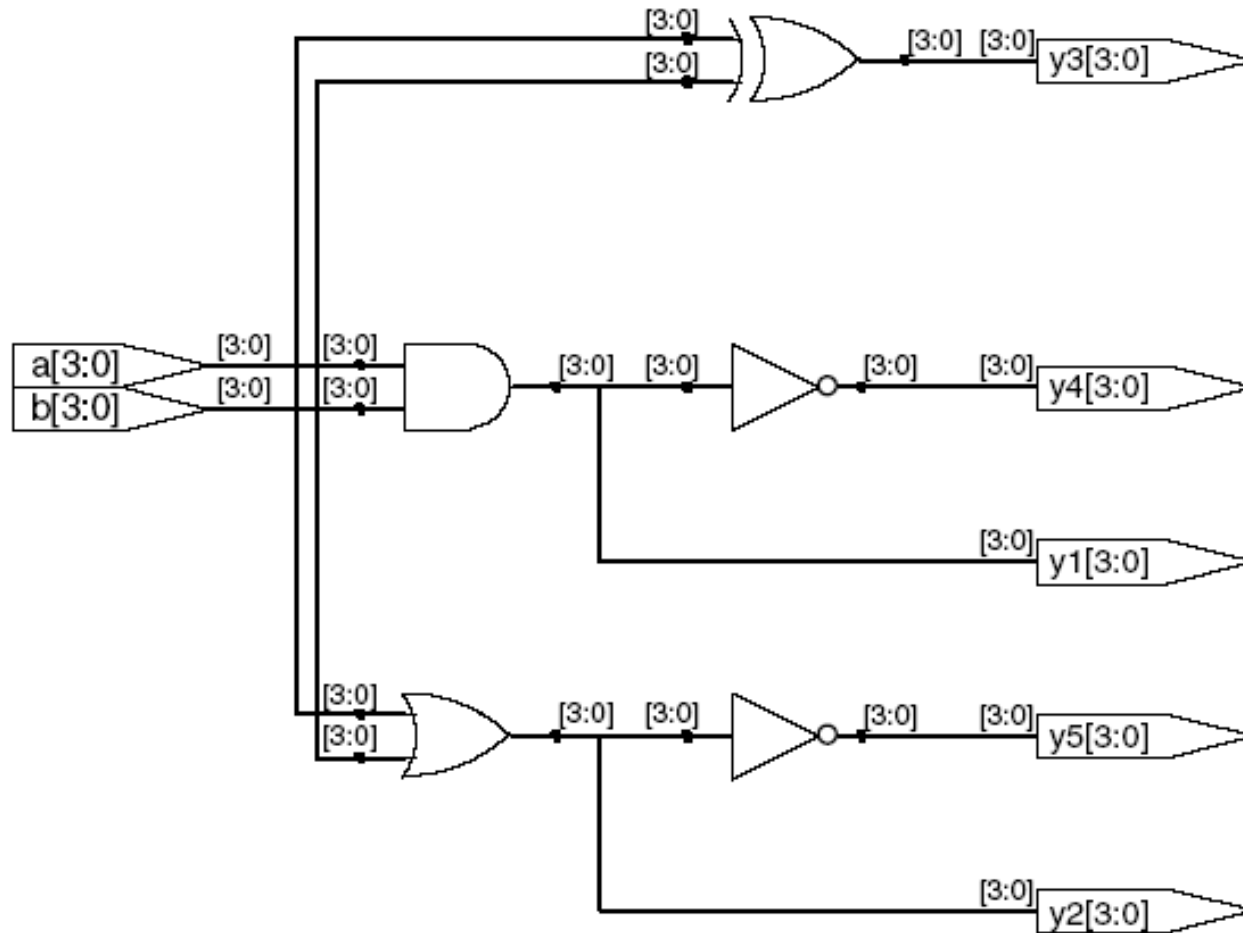
```
module gates(input  [3:0]  a, b,  
              output [3:0] y1, y2, y3, y4, y5);
```

```
    /* Five different two-input logic  
       gates acting on 4 bit buses */
```

```
    assign y1 = a & b;      // AND  
    assign y2 = a | b;      // OR  
    assign y3 = a ^ b;      // XOR  
    assign y4 = ~(a & b);   // NAND  
    assign y5 = ~(a | b);   // NOR
```

```
endmodule
```

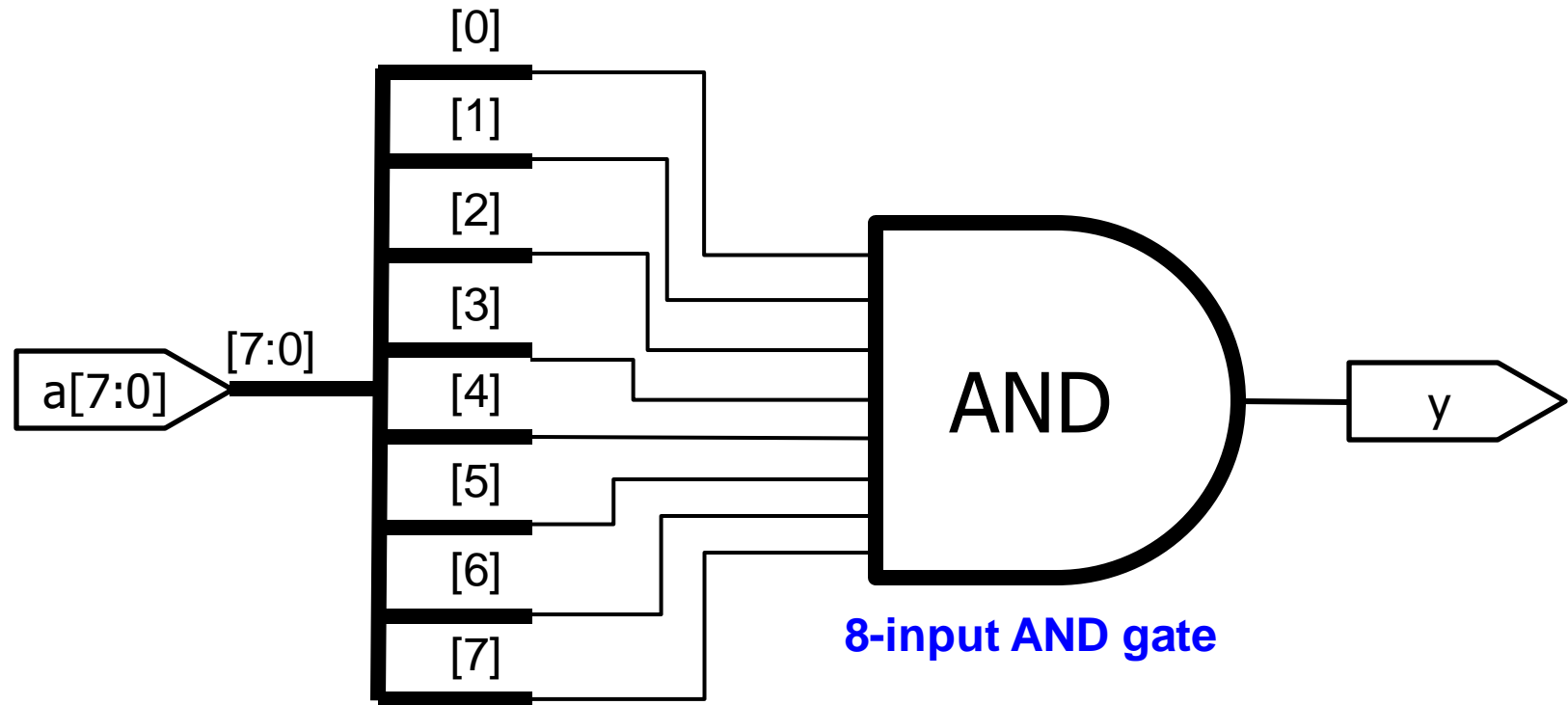
Bitwise Operators: Schematic View



Reduction Operators in Behavioral Verilog

```
module and8(input  [7:0] a,  
            output  y);  
  
    assign y = &a;  
  
    // &a is much easier to write than  
    // assign y = a[7] & a[6] & a[5] & a[4] &  
    //             a[3] & a[2] & a[1] & a[0];  
  
endmodule
```


Reduction Operators: Schematic View

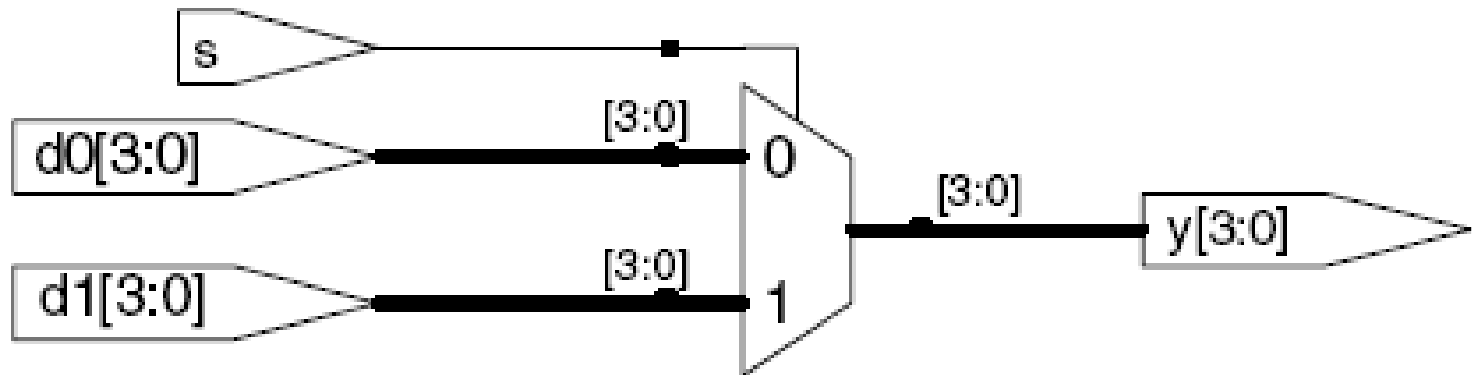


Conditional Assignment in Behavioral Verilog

```
module mux2(input  [3:0] d0, d1,  
            input      s,  
            output [3:0] y);  
  
    assign y = s ? d1 : d0;  
    // if (s) then y=d1 else y=d0;  
  
endmodule
```

- ? : is also called a **ternary operator** as it operates on three inputs:
 - ❑ s
 - ❑ d1
 - ❑ d0

Conditional Assignment: Schematic View



More Complex Conditional Assignments

```
module mux4(input  [3:0] d0, d1, d2, d3
            input  [1:0] s,
            output [3:0] y);

    assign y = s[1] ? ( s[0] ? d3 : d2)
                : ( s[0] ? d1 : d0);

    // if (s1) then
    //     if (s0) then y=d3 else y=d2
    // else
    //     if (s0) then y=d1 else y=d0

endmodule
```

Even More Complex Conditional Assignments

```
module mux4(input  [3:0] d0, d1, d2, d3
            input  [1:0] s,
            output [3:0] y);

    assign y = (s == 2'b11) ? d3 :
               (s == 2'b10) ? d2 :
               (s == 2'b01) ? d1 :
               d0;

    // if      (s = "11" ) then y= d3
    // else if (s = "10" ) then y= d2
    // else if (s = "01" ) then y= d1
    // else                      y= d0

endmodule
```

Precedence of Operations in Verilog

Highest

~	NOT
*, /, %	mult, div, mod
+, -	add, sub
<<, >>	shift
<<<, >>>	arithmetic shift
<, <=, >, >=	comparison
==, !=	equal, not equal
&, ~&	AND, NAND
^, ~^	XOR, XNOR
, ~	OR, NOR
?:	ternary operator

Lowest

How to Express Numbers ?

N' Bxx

8' b0000_0001

- (N) Number of bits

- Expresses how many bits will be used to store the value

- (B) Base


- Can be b (binary), h (hexadecimal), d (decimal), o (octal)

- (xx) Number

- The value expressed in base
- Can also have X (invalid) and Z (floating), as values
- Underscore _ can be used to improve readability

Number Representation in Verilog

Verilog	Stored Number	Verilog	Stored Number
4'b1001	1001	4'd5	0101
8'b1001	0000 1001	12'hFA3	1111 1010 0011
8'b0000_1001	0000 1001	8'o12	00 001 010
8'bxX0X1zZ1	XX0X 1ZZ1	4'h7	0111
'b01	0000 .. 0001	12'h0	0000 0000 0000

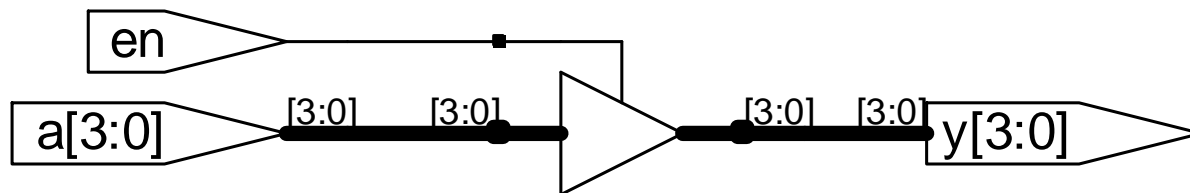


**32 bits
(default)**

Reminder: Floating Signals (Z)

- **Floating signal:** Signal that is not driven by any circuit
 - Open circuit, floating wire
- Also known as: **high impedance, hi-Z, tri-stated** signals

```
module tristate_buffer(input  [3:0] a,  
                      input      en,  
                      output [3:0] y);  
  
    assign y = en ? a : 4'bz;  
  
endmodule
```



Tri-State Buffer

- A tri-state buffer enables gating of different signals onto a wire

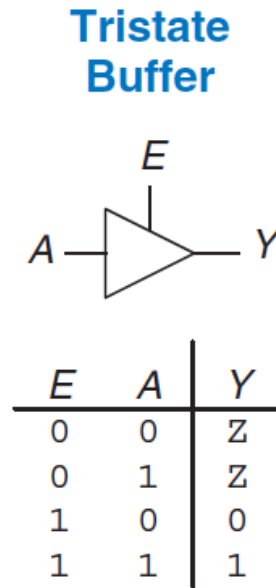


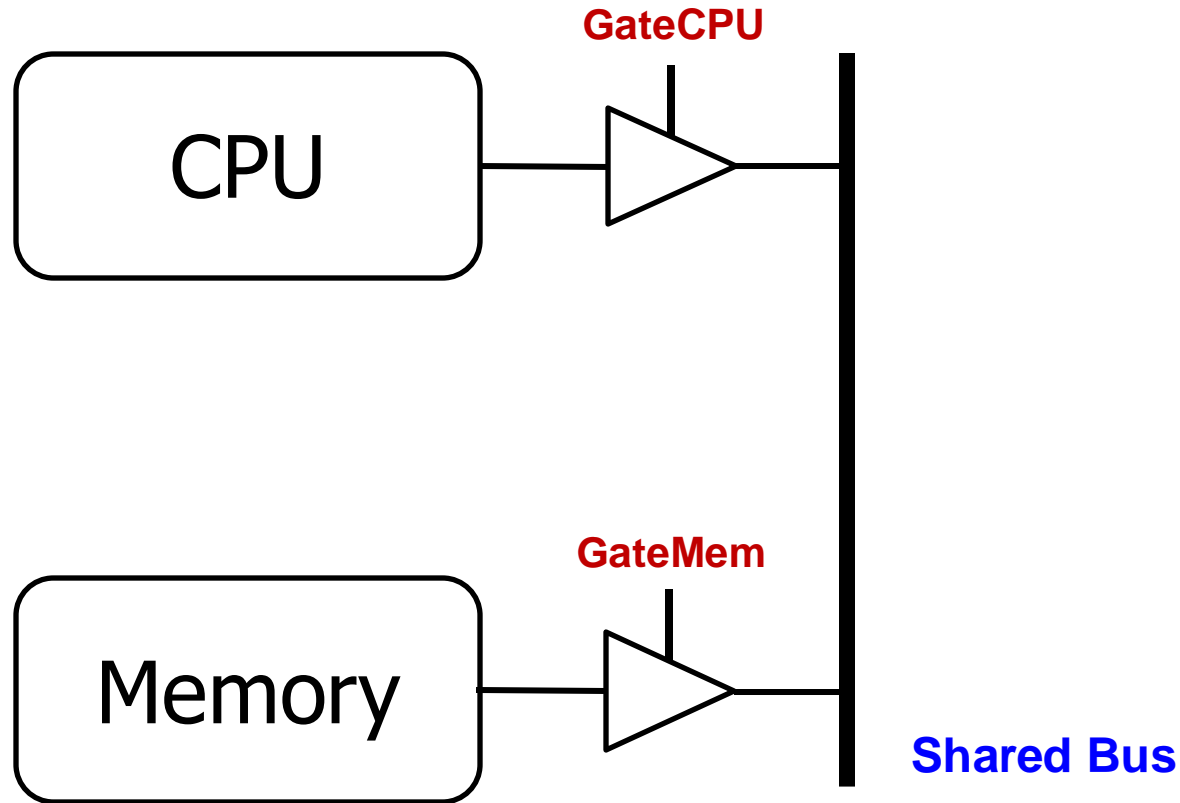
Figure 2.40 Tristate buffer

- **Floating signal (Z):** Signal that is not driven by any circuit
 - Open circuit, floating wire

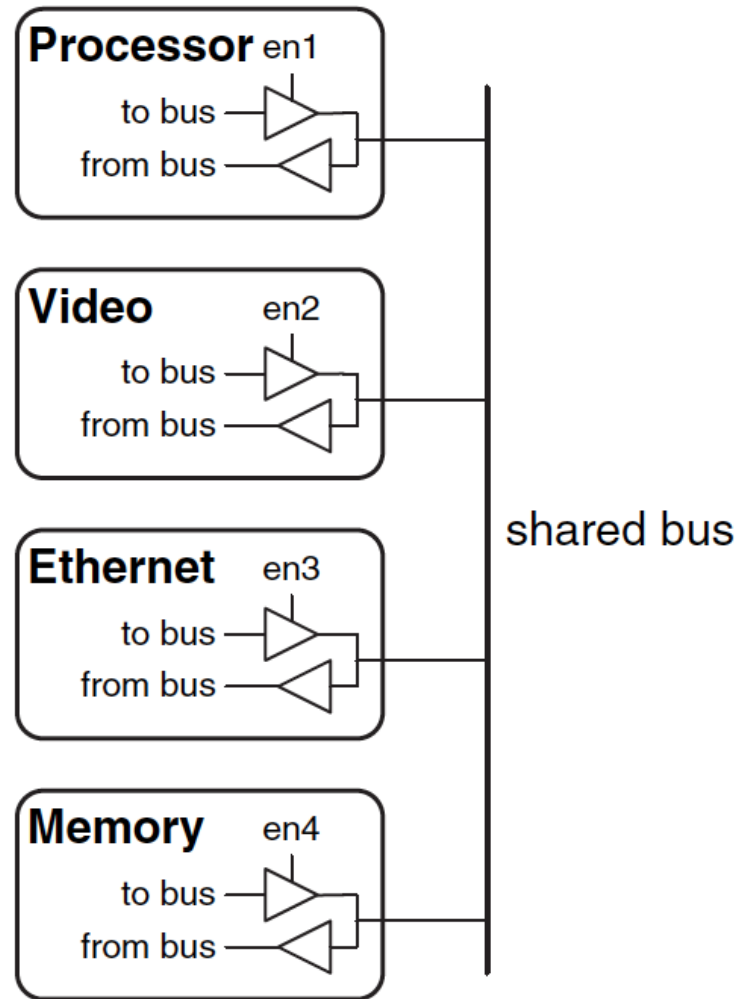
Example: Use of Tri-State Buffers

- Imagine a wire connecting the CPU and memory
 - At any time only the CPU or the memory can place a value on the wire, both not both
 - You can have two tri-state buffers: one driven by CPU, the other memory; and ensure at most one is enabled at any time

Example Design with Tri-State Buffers



Another Example



Truth Table for AND with Z and X

AND		A			
		0	1	Z	X
B	0	0	0	0	0
	1	0	1	X	X
	Z	0	X	X	X
	X	0	X	X	X

What Happens with HDL Code?

■ Synthesis

- ❑ Modern tools are able to **map** ***synthesizable** HDL code* into low-level *cell libraries* → *netlist describing gates and wires*
- ❑ They can perform many **optimizations**
- ❑ ... however they **can not guarantee** that a solution is optimal
 - Mainly due to **computationally expensive** **placement** and **routing** algorithms
- ❑ Most common way of Digital Design these days

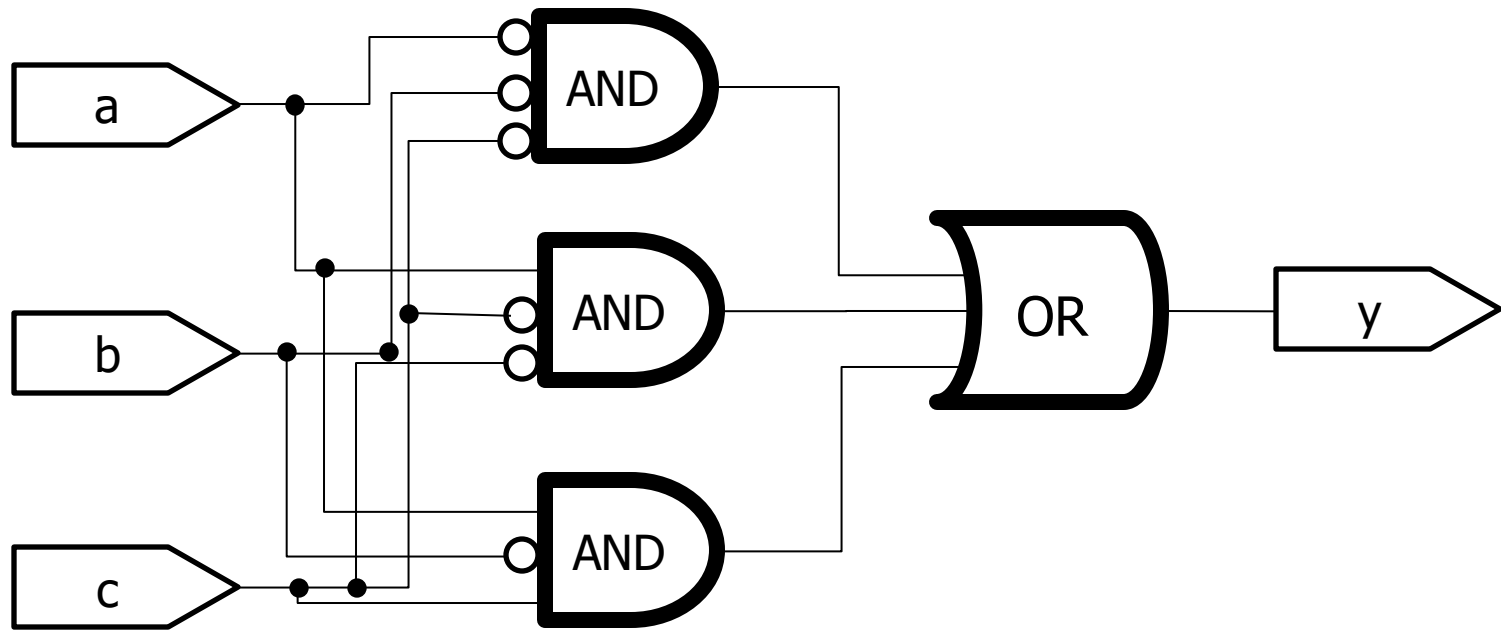
■ Simulation

- ❑ Allows the behavior of the circuit to be **verified without actually manufacturing the circuit**
- ❑ Simulators can work on *structural* or *behavioral* HDL

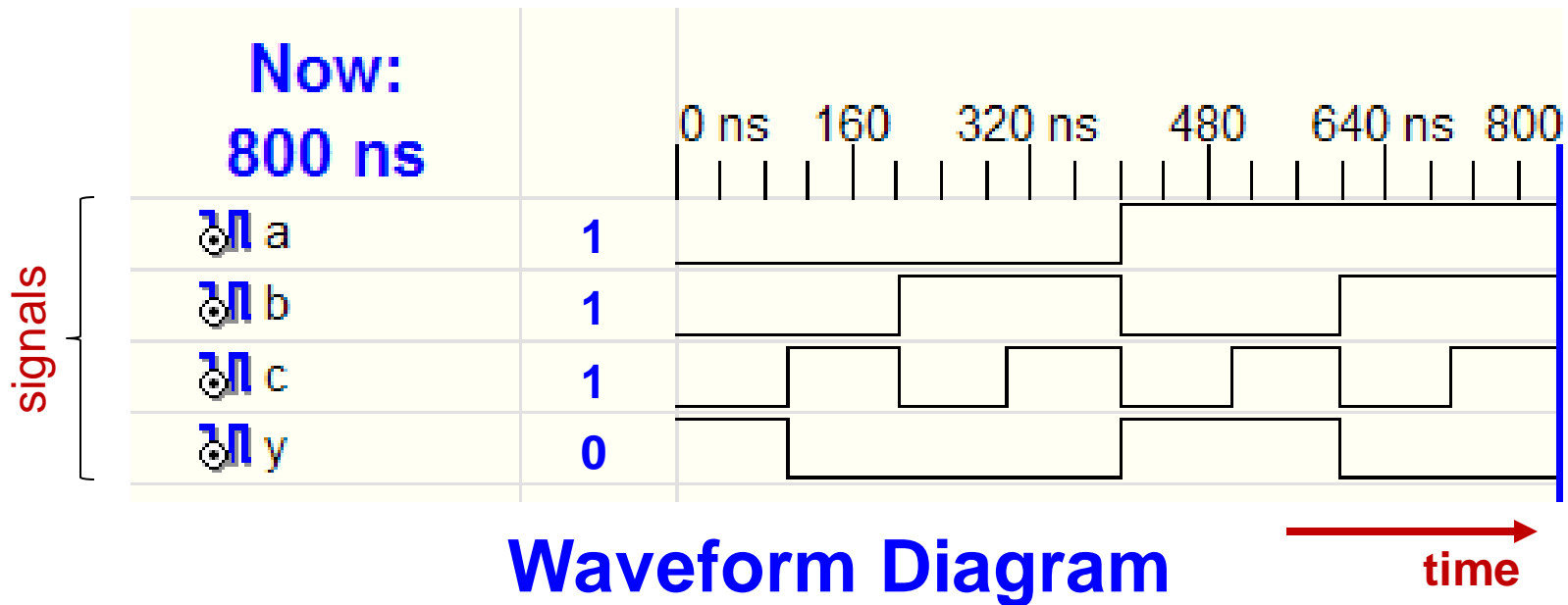
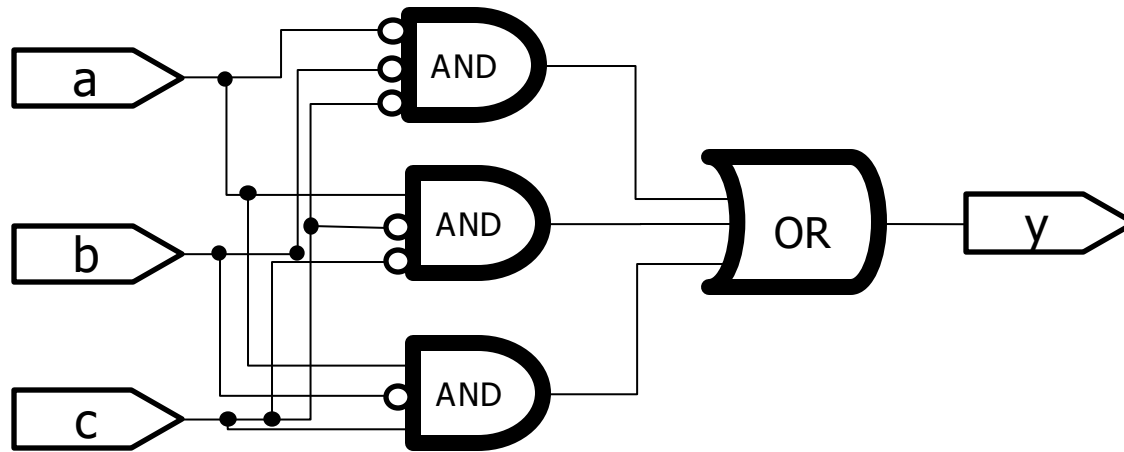
Recall This “example”

```
module example (a, b, c, y);  
    input a;  
    input b;  
    input c;  
    output y;  
  
    // here comes the circuit description  
    assign y = ~a & ~b & ~c |  
               a & ~b & ~c |  
               a & ~b & c;  
  
endmodule
```


Synthesizing the “example”



Simulating the “example”



What We Have Seen So Far

- **Describing structural hierarchy with Verilog**
 - Instantiate modules in an other module
- **Describing functionality using behavioral modeling**
- **Writing simple logic equations**
 - We can write AND, OR, XOR, ...
- **Multiplexer functionality**
 - If ... then ... else
- **We can describe constants**
- **But there is more...**

More Verilog Examples

- We can write Verilog code in many different ways
- Let's see how we can express the same functionality by developing Verilog code
 - At a low-level of abstraction
 - Poor readability
 - More optimization opportunities (especially for low-level tools)
 - At a high-level of abstraction
 - Better readability
 - Limited optimization opportunities

Comparing Two Numbers

- **Defining your own gates as new modules**
- We will use our gates to show the different ways of implementing a 4-bit comparator (equality checker)

An XNOR gate

```
module MyXnor (input A, B,  
               output Z);  
  
    assign Z = ~(A ^ B); //not XOR  
  
endmodule
```

An AND gate

```
module MyAnd (input A, B,  
              output Z);  
  
    assign Z = A & B;    // AND  
  
endmodule
```

Gate-Level Implementation

```
module compare (input a0, a1, a2, a3, b0, b1, b2, b3,
                output eq);
    wire c0, c1, c2, c3, c01, c23;

    MyXnor i0 (.A(a0), .B(b0), .Z(c0) ); // XNOR
    MyXnor i1 (.A(a1), .B(b1), .Z(c1) ); // XNOR
    MyXnor i2 (.A(a2), .B(b2), .Z(c2) ); // XNOR
    MyXnor i3 (.A(a3), .B(b3), .Z(c3) ); // XNOR
    MyAnd haha (.A(c0), .B(c1), .Z(c01) ); // AND
    MyAnd hoho (.A(c2), .B(c3), .Z(c23) ); // AND
    MyAnd bubu (.A(c01), .B(c23), .Z(eq) ); // AND

endmodule
```

Using Logical Operators

```
module compare (input a0, a1, a2, a3, b0, b1, b2, b3,
                 output eq);
    wire c0, c1, c2, c3, c01, c23;

    MyXnor i0 (.A(a0), .B(b0), .Z(c0) ); // XNOR
    MyXnor i1 (.A(a1), .B(b1), .Z(c1) ); // XNOR
    MyXnor i2 (.A(a2), .B(b2), .Z(c2) ); // XNOR
    MyXnor i3 (.A(a3), .B(b3), .Z(c3) ); // XNOR
    assign c01 = c0 & c1;
    assign c23 = c2 & c3;
    assign eq  = c01 & c23;

endmodule
```

Eliminating Intermediate Signals

```
module compare (input a0, a1, a2, a3, b0, b1, b2, b3,
                output eq);
    wire c0, c1, c2, c3;

    MyXnor i0 (.A(a0), .B(b0), .Z(c0) ); // XNOR
    MyXnor i1 (.A(a1), .B(b1), .Z(c1) ); // XNOR
    MyXnor i2 (.A(a2), .B(b2), .Z(c2) ); // XNOR
    MyXnor i3 (.A(a3), .B(b3), .Z(c3) ); // XNOR
    // assign c01 = c0 & c1;
    // assign c23 = c2 & c3;
    // assign eq  = c01 & c23;
    assign eq    = c0 & c1 & c2 & c3;

endmodule
```


Multi-Bit Signals (Bus)

```
module compare (input [3:0] a, input [3:0] b,  
                output eq);  
    wire [3:0] c; // bus definition  
  
    MyXnor i0 (.A(a[0]), .B(b[0]), .Z(c[0]) ); // XNOR  
    MyXnor i1 (.A(a[1]), .B(b[1]), .Z(c[1]) ); // XNOR  
    MyXnor i2 (.A(a[2]), .B(b[2]), .Z(c[2]) ); // XNOR  
    MyXnor i3 (.A(a[3]), .B(b[3]), .Z(c[3]) ); // XNOR  
  
    assign eq  = &c; // short format  
  
endmodule
```

Bitwise Operations

```
module compare (input [3:0] a, input [3:0] b,  
                output eq);  
    wire [3:0] c; // bus definition  
  
    // MyXnor i0 (.A(a[0]), .B(b[0]), .Z(c[0]) );  
    // MyXnor i1 (.A(a[1]), .B(b[1]), .Z(c[1]) );  
    // MyXnor i2 (.A(a[2]), .B(b[2]), .Z(c[2]) );  
    // MyXnor i3 (.A(a[3]), .B(b[3]), .Z(c[3]) );  
  
    assign c = ~(a ^ b); // XNOR  
  
    assign eq  = &c; // short format  
  
endmodule
```

Highest Abstraction Level: Comparing Two Numbers

```
module compare (input [3:0] a, input [3:0] b,  
                output eq);
```

```
// assign c = ~(a ^ b); // XNOR
```

```
// assign eq = &c; // short format
```

```
assign eq = (a == b) ? 1 : 0; // really short
```

```
endmodule
```

Writing More Reusable Verilog Code

- We have a module that can compare two 4-bit numbers
- What if in the overall design we need to compare:
 - **5**-bit numbers?
 - **6**-bit numbers?
 - ...
 - **N**-bit numbers?
 - Writing code for each case looks tedious
- What could be a better way?

Parameterized Modules

In Verilog, we can define **module parameters**

```
module mux2
  #(parameter width = 8)  // name and default value
  (input  [width-1:0] d0, d1,
   input                                     s,
   output [width-1:0] y);

  assign y = s ? d1 : d0;
endmodule
```

We can set the parameters to different values
when instantiating the module

Instantiating Parameterized Modules

```
module mux2
  #(parameter width = 8) // name and default value
  (input [width-1:0] d0, d1,
   input          s,
   output [width-1:0] y);

  assign y = s ? d1 : d0;
endmodule
```

What About Timing?

- It is possible to define *timing relations* in Verilog. **BUT:**
 - These are **ONLY** for simulation
 - They **CAN NOT** be synthesized
 - They are used for *modeling delays* in a circuit

```
'timescale 1ns/1ps
module simple (input a, output z1, z2);

assign #5 z1 = ~a; // inverted output after 5ns
assign #9 z2 = a;  // output after 9ns

endmodule
```

More to come later today!

Good Practices

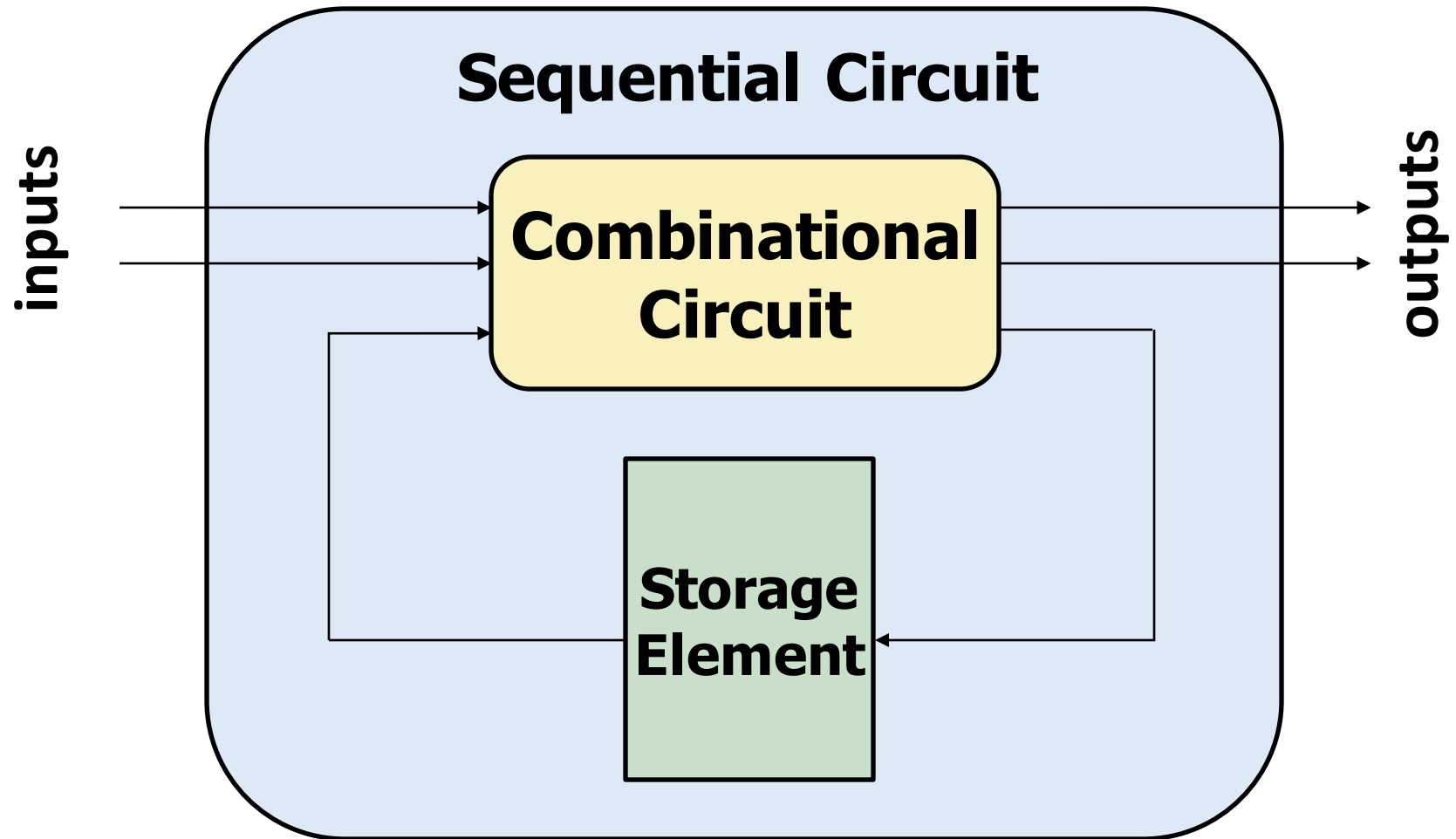
- Develop/use a **consistent** naming style
- Use **MSB to LSB ordering** for buses
 - Use “**a[31:0]**”, **not** “**a[0:31]**”
- Define **one module per file**
 - Makes managing your design hierarchy easier
- Use a file name that equals module name
 - e.g., module **TryThis** is defined in a file called **TryThis.v**
- Always keep in mind that **Verilog describes hardware**

Summary (HDL for Combinational Logic)

- We have seen an overview of Verilog
- Discussed structural and behavioral modeling
- Studied combinational logic constructs

Implementing Sequential Logic Using Verilog

Combinational + Memory = Sequential



Sequential Logic in Verilog

- Define blocks that have memory
 - *Flip-Flops, Latches, Finite State Machines*
- Sequential Logic state transition is triggered by a "CLOCK" signal
 - Latches are sensitive to level of the signal
 - Flip-flops are sensitive to the transitioning of signal
- Combinational HDL constructs are **not** sufficient to express sequential logic
 - We need **new constructs**:
 - **always**
 - **posedge/negedge**

The “always” Block

```
always @ (sensitivity list)  
    statement;
```

Whenever the event in the **sensitivity list** occurs,
the statement is **executed**

Example: D Flip-Flop

```
module flop(input          clk,
            input    [3:0] d,
            output reg [3:0] q);

    always @ (posedge clk)
        q <- d;                // pronounced "q gets d"

endmodule
```

- **posedge** defines a rising edge (transition from 0 to 1).
- Statement executed when the **clk signal rises (posedge of clk)**
- Once the clk signal rises: the value of **d** is copied to **q**

Example: D Flip-Flop

```
module flop(input          clk,
            input    [3:0] d,
            output reg [3:0] q);

    always @ (posedge clk)
        q <= d;                // pronounced “q gets d”

endmodule
```

- **assign** statement is **not** used within an always block
- **<=** describes a **non-blocking** assignment
 - We will see the difference between **blocking assignment** and **non-blocking** assignment soon

Example: D Flip-Flop

```
module flop(input          clk,  
            input  [3:0] d,  
            output reg [3:0] q);  
  
    always @ (posedge clk)  
        q <= d;                // pronounced “q gets d”  
  
endmodule
```

- Assigned variables need to be declared as **reg**
- The name **reg** does not necessarily mean that the value is a register (It could be, but it does not have to be)
- We will see examples later

Asynchronous and Synchronous Reset

- **Reset** signals are used to **initialize** the hardware to a known state
 - Usually activated **at system start** (on power up)
- **Asynchronous Reset**
 - The reset signal is sampled **independent** of the clock
 - Reset gets the highest priority
 - Sensitive to **glitches**, may have **metastability** issues
 - Will be discussed in Lecture 8
- **Synchronous Reset**
 - The reset signal is sampled **with respect to the clock**
 - The reset **should be active long enough** to get sampled at the clock edge
 - Results in **completely synchronous circuit**

D Flip-Flop with Asynchronous Reset

```
module flop_ar (input          clk,
                input          reset,
                input    [3:0] d,
                output reg [3:0] q);

  always @ (posedge clk, negedge reset)
  begin
    if (reset == 0) q <= 0;    // when reset
    else           q <= d;    // when clk
  end
endmodule
```

- In this example: two events can trigger the process:
 - A **rising edge** on clk
 - A **falling edge** on reset

D Flip-Flop with Asynchronous Reset

```
module flop_ar (input          clk,
                input          reset,
                input    [3:0] d,
                output reg [3:0] q);

    always @ (posedge clk, negedge reset)
    begin
        if (reset == 0) q <= 0;    // when reset
        else            q <= d;    // when clk
    end
endmodule
```

- For longer statements, a **begin-end** pair can be used
 - ❑ To improve readability
 - ❑ In this example, it was not necessary, but it is a good idea

D Flip-Flop with Asynchronous Reset

```
module flop_ar (input          clk,
                input          reset,
                input    [3:0] d,
                output reg [3:0] q);

    always @ (posedge clk, negedge reset)
    begin
        if (reset == 0) q <= 0; // when reset
        else            q <= d;  // when clk
    end
endmodule
```

- First **reset** is checked: if **reset** is 0, **q** is set to 0.
 - This is an **asynchronous** reset as the reset can happen **independently** of the clock (on the negative edge of reset signal)
- If there is no reset, then regular assignment takes effect

D Flip-Flop with Synchronous Reset

```
module flop_sr (input          clk,
                input          reset,
                input    [3:0] d,
                output reg [3:0] q);

    always @ (posedge clk)
    begin
        if (reset == '0') q <= 0;    // when reset
        else               q <= d;    // when clk
    end
endmodule
```

- The process is sensitive to only clock
 - Reset *happens only* when the *clock rises*. This is a *synchronous* reset

D Flip-Flop with Enable and Reset

```
module flop_en_ar (input          clk,
                  input          reset,
                  input          en,
                  input [3:0] d,
                  output reg [3:0] q);

  always @ (posedge clk, negedge reset)
  begin
    if (reset == '0') q <= 0;    // when reset
    else if (en)      q <= d;    // when en AND clk
  end
endmodule
```

- A flip-flop with **enable** and **reset**
 - Note that the **en** signal is **not** in the *sensitivity list*
- **q** gets **d** only when **clk** is rising **and** **en** is 1

Example: D Latch

```
module latch (input          clk,
              input          [3:0] d,
              output reg [3:0] q);

    always @ (clk, d)
        if (clk) q <= d;           // latch is transparent when
                                   // clock is 1

endmodule
```

Summary: Sequential Statements So Far

- Sequential statements are within an `always` block
- The sequential block is triggered with a change in the `sensitivity list`
- Signals assigned within an **`always`** must be declared as `reg`
- We use `<=` for (non-blocking) assignments and do not use `assign` within the `always` block.

Basics of **always** Blocks

```
module example (input          clk,
                input    [3:0] d,
                output reg [3:0] q);

    wire [3:0] normal;           // standard wire
    reg  [3:0] special;          // assigned in always

    always @ (posedge clk)
        special <= d;            // first FF array

    assign normal = ~ special;    // simple assignment

    always @ (posedge clk)
        q <= normal;             // second FF array
endmodule
```

You can have as many **always** blocks as needed

Assignment to the same signal in different always blocks is not allowed!

Why Does an **always** Block Remember?

```
module flop (input          clk,  
             input    [3:0] d,  
             output reg [3:0] q);  
  
    always @ (posedge clk)  
        begin  
            q <= d;    // when clk rises copy d to q  
        end  
endmodule
```

- This statement describes what happens to signal **q**
- ... but what happens when the clock is not rising?
- The value of **q** is preserved (remembered)

An **always** Block Does **NOT** Always Remember

```
module comb (input          inv,
              input    [3:0] data,
              output reg [3:0] result);

    always @ (inv, data)          // trigger with inv, data
        if (inv) result <= ~data; // result is inverted data
        else    result <= data;   // result is data

endmodule
```

- This statement describes what happens to signal **result**
 - When **inv** is 1, **result** is **~data**
 - When **inv** is not 1, **result** is **data**
- The circuit is combinational (no memory)
 - **result** is assigned a value in all cases of the **if .. else** block, always

always Blocks for Combinational Circuits

- An **always** block defines **combinational logic** if:
 - All outputs are always (**continuously**) updated
 1. All right-hand side signals are in the sensitivity list
 - You can use **always @*** for short
 2. All left-hand side signals get assigned in every possible condition of **if .. else** and **case** blocks
- It is easy to make mistakes and **unintentionally describe memorizing elements** (latches)
 - **Vivado** will most likely warn you. Make sure you check the warning messages
- **Always** blocks allow powerful combinational logic statements
 - **if .. else**
 - **case**

Sequential or Combinational?

```
wire enable, data;
reg out_a, out_b;

always @ (*) begin
    out_a = 1'b0;
    if(enable) begin
        out_a = data;
        out_b = data;
    end
end
```

No assignment for ~enable

Sequential

```
wire enable, data;
reg out_a, out_b;

always @ (data) begin
    out_a = 1'b0;
    out_b = 1'b0;
    if(enable) begin
        out_a = data;
        out_b = data;
    end
end
```

Not in the sensitivity list

Sequential

The **always** Block is **NOT** Always Practical/Nice

```
reg [31:0] result;
wire [31:0] a, b, comb;
wire      sel,

always @ (a, b, sel)    // trigger with a, b, sel
    if (sel) result <= a; // result is a
    else      result <= b; // result is b

assign comb = sel ? a : b;
```

- Both statements describe the **same** multiplexer
- In this case, the **always** block is more work

always Block for Case Statements (Handy!)

```
module sevenssegment (input      [3:0] data,
                      output reg [6:0] segments);

  always @ ( * )                // * is short for all signals
  case (data)                   // case statement
    4'd0: segments = 7'b111_1110; // when data is 0
    4'd1: segments = 7'b011_0000; // when data is 1
    4'd2: segments = 7'b110_1101;
    4'd3: segments = 7'b111_1001;
    4'd4: segments = 7'b011_0011;
    4'd5: segments = 7'b101_1011;
    // etc etc
    default: segments = 7'b000_0000; // required
  endcase

endmodule
```

Summary: **always** Block

- `if .. else` can only be used in `always` blocks
- The `always` block is **combinational** only if all `regs` within the block are always assigned to a signal
 - Use the `default` case to make sure you do not forget an unimplemented case, which may otherwise result in a latch
- Use `case` statement to be able to check for don't cares

Non-Blocking and Blocking Assignments

Non-blocking (<=)

```
always @ (a)
begin
    a <= 2'b01;
    b <= a;
// all assignments are made here
// b is not (yet) 2'b01
end
```

- All assignments are made at the end of the block
- All assignments are made in parallel, process flow is **not-blocked**

Blocking (=)

```
always @ (a)
begin
    a = 2'b01;
// a is 2'b01
    b = a;
// b is now 2'b01 as well
end
```

- Each assignment is made immediately
- Process waits until the first assignment is complete, it **blocks** progress

Example: Blocking Assignment

- Assume all inputs are initially '0'

```
always @ ( * )  
begin  
    p    = a ^ b ;           // p    = 0    1  
    g    = a & b ;           // g    = 0    0  
    s    = p ^ cin ;        // s    = 0    1  
    cout = g | (p & cin) ;  // cout = 0    0  
end
```

- If **a** changes to '1'
 - All values are updated in order

The Same Example: Non-Blocking Assignment

- Assume all inputs are initially '0'

```
always @ ( * )
begin
    p    <= a ^ b ;           // p    = 0  1
    g    <= a & b ;           // g    = 0  0
    s    <= p ^ cin ;         // s    = 0  0
    cout <= g | (p & cin) ;    // cout = 0  0
end
```

- If **a** changes to '1'
 - All assignments are concurrent
 - When **s** is being assigned, **p** is still 0

The Same Example: Non-Blocking Assignment

- After the first iteration, **p** has changed to '1' as well

```
always @ ( * )  
begin  
    p    <= a ^ b ;           // p    = 1    1  
    g    <= a & b ;           // g    = 0    0  
    s    <= p ^ cin ;         // s    = 0    1  
    cout <= g | (p & cin) ;   // cout = 0    0  
end
```

- Since there is a change in **p**, the process **triggers again**
- This time **s** is calculated with **p=1**

Rules for Signal Assignment

- Use `always @(posedge clk)` and `non-blocking` assignments (`<=`) to model `synchronous sequential logic`

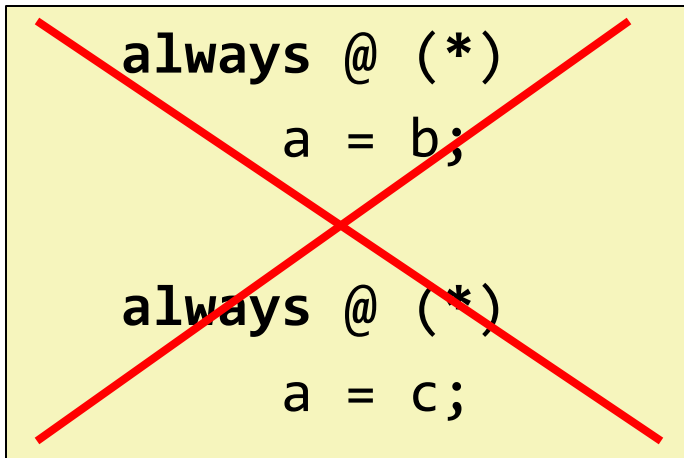
```
always @ (posedge clk)
    q <= d; // non-blocking
```

- Use continuous assignments (`assign`) to model simple combinational logic

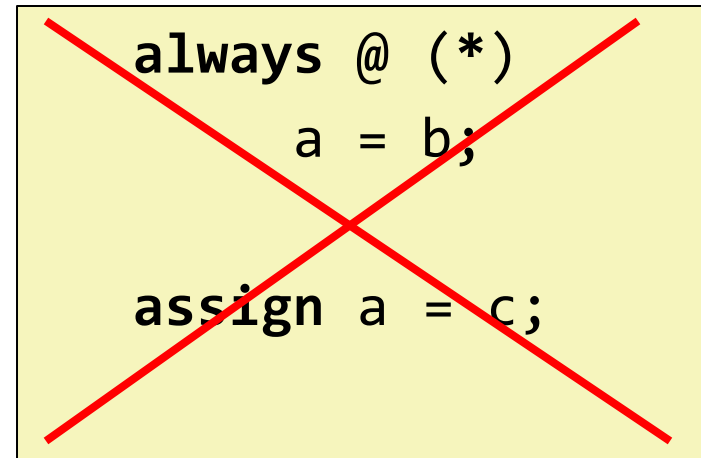
```
assign y = a & b;
```

Rules for Signal Assignment (Cont.)

- Use **always @ (*)** and **blocking** assignments (=) to model more **complicated combinational logic**.
- You **cannot** make assignments to the **same** signal in more than one always block or in a *continuous assignment*



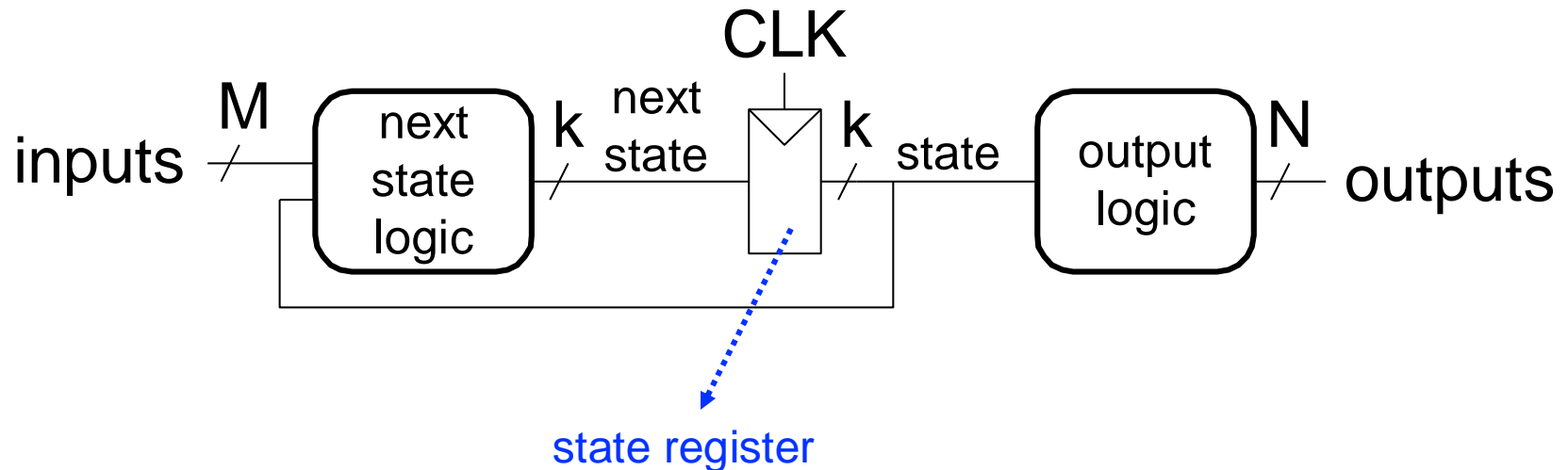
```
always @ (*)  
    a = b;  
  
always @ (*)  
    a = c;
```



```
always @ (*)  
    a = b;  
  
assign a = c;
```

Recall: Finite State Machines (FSMs)

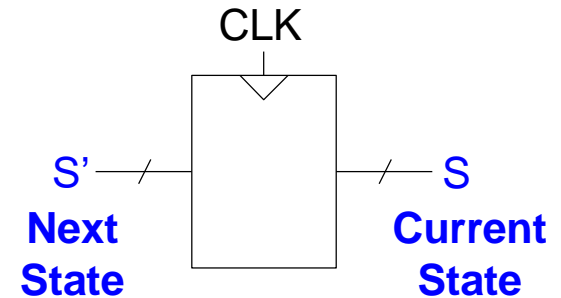
- Each FSM consists of three separate parts:
 - next state logic
 - state register
 - output logic



Recall: Finite State Machines (FSMs) Comprise

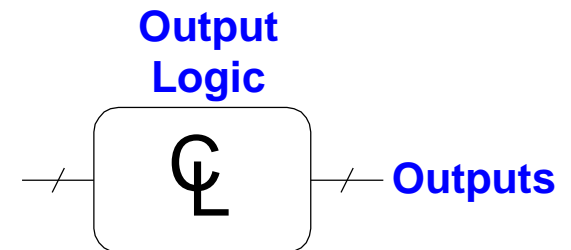
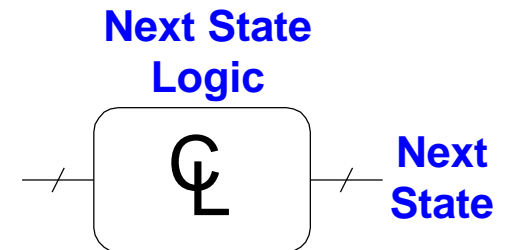
■ Sequential circuits

- State register(s)
 - Store the current state and
 - Load the next state at the clock edge

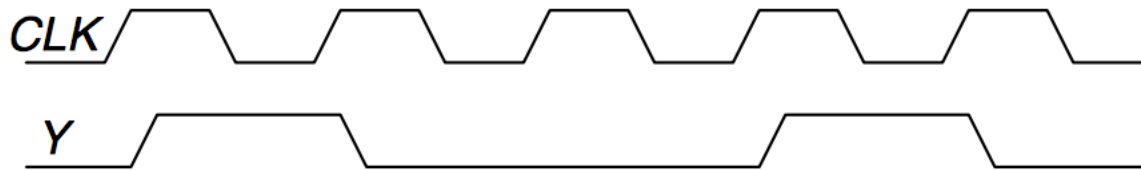


■ Combinational Circuits

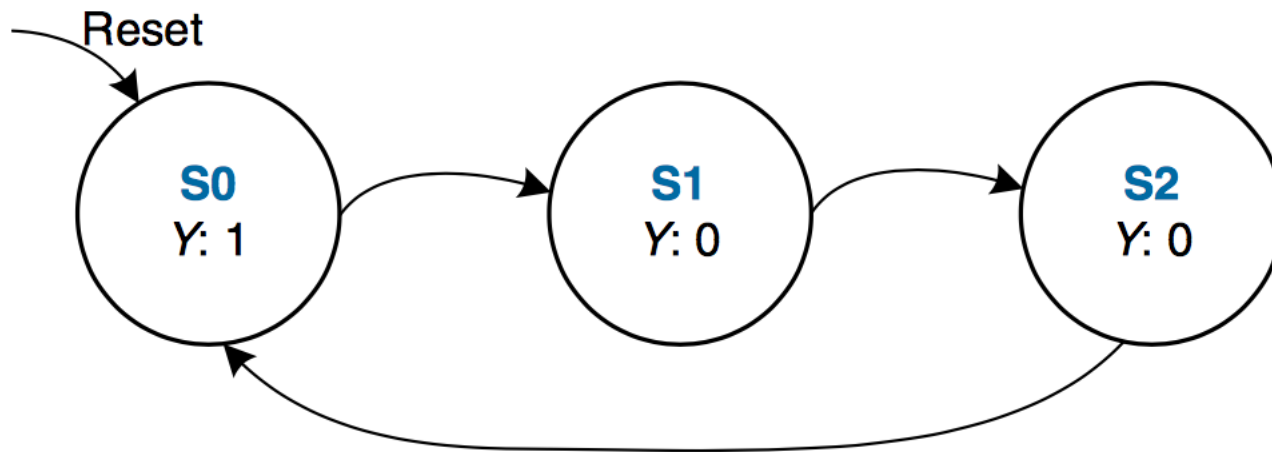
- Next state logic
 - Determines what the next state will be
- Output logic
 - Generates the outputs



FSM Example 1: Divide the Clock Frequency by 3



The output Y is HIGH for **one clock cycle out of every 3**. In other words, the output **divides the frequency of the clock by 3**.

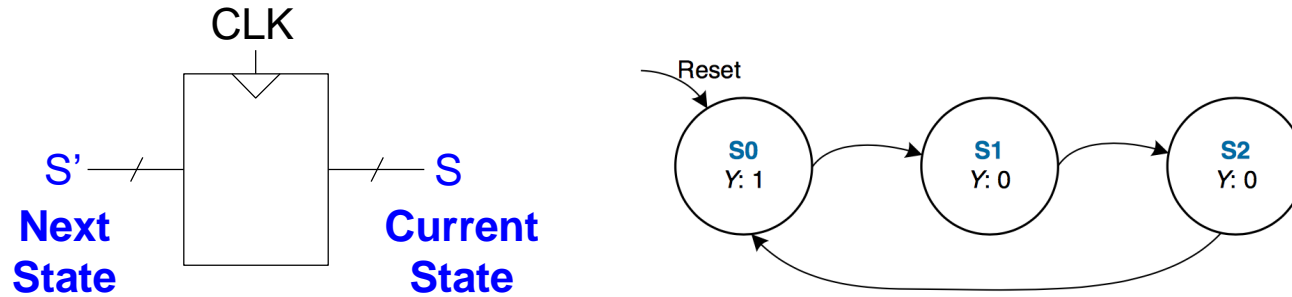


Implementing FSM Example 1: Definitions

```
module divideby3FSM (input clk,  
                    input reset,  
                    output q);  
  
    reg [1:0] state, nextstate;  
  
    parameter S0 = 2'b00;  
    parameter S1 = 2'b01;  
    parameter S2 = 2'b10;
```

- We define **state** and **nextstate** as 2-bit reg
- The parameter descriptions are **optional**, it makes reading easier

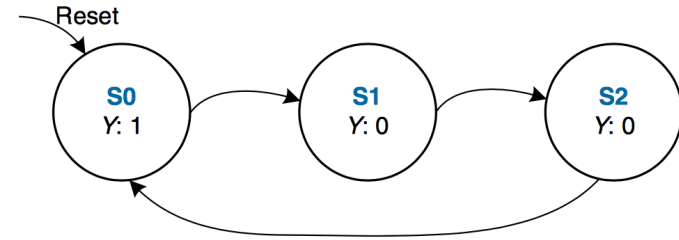
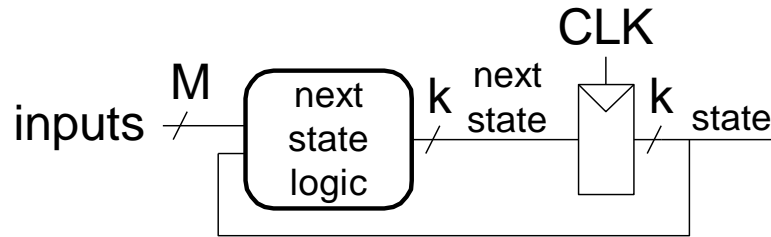
Implementing FSM Example 1: State Register



```
// state register
always @ (posedge clk, posedge reset)
  if (reset) state <= S0;
  else      state <= nextstate;
```

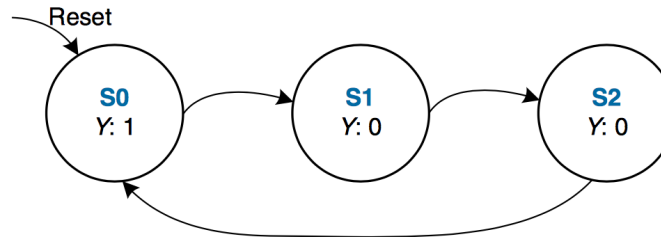
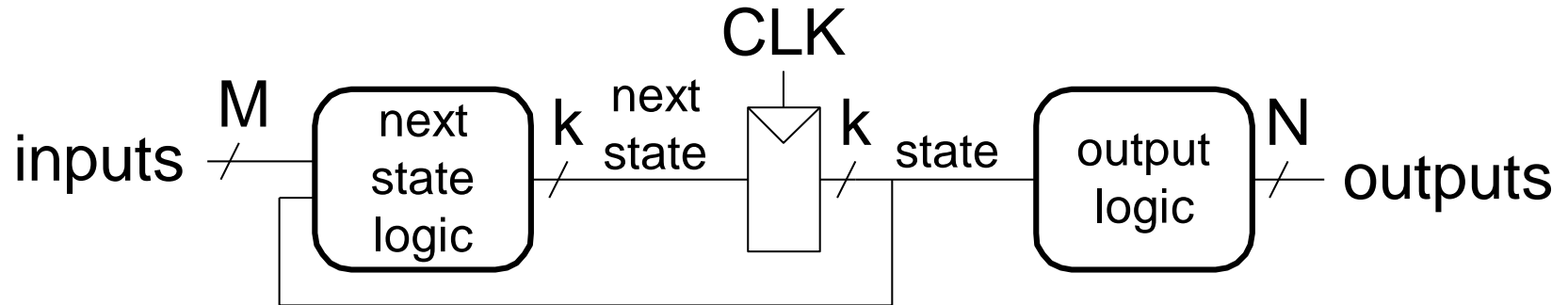
- This part defines the **state register** (memorizing process)
- Sensitive to only **clk**, **reset**
- In this example, **reset** is active when it is '1' (active-high)

Implementing FSM Example 1: Next State Logic



```
// next state logic
always @ (*)
  case (state)
    S0:      nextstate = S1;
    S1:      nextstate = S2;
    S2:      nextstate = S0;
    default: nextstate = S0;
  endcase
```

Implementing FSM Example 1: Output Logic



```
// output logic  
assign q = (state == S0);
```

- In this example, output depends only on state
 - **Moore type FSM**

Implementation of FSM Example 1

```
module divideby3FSM (input clk, input reset, output q);
    reg [1:0] state, nextstate;

    parameter S0 = 2'b00; parameter S1 = 2'b01; parameter S2 = 2'b10;

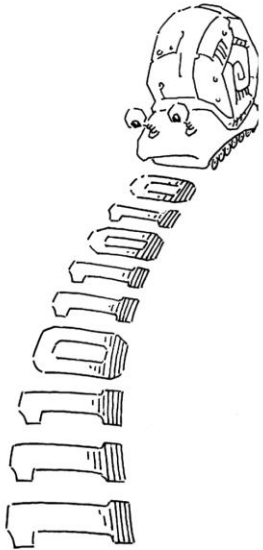
    always @ (posedge clk, posedge reset) // state register
        if (reset) state <= S0;
        else      state <= nextstate;

    always @ (*) // next state logic
        case (state)
            S0:      nextstate = S1;
            S1:      nextstate = S2;
            S2:      nextstate = S0;
            default: nextstate = S0;
        endcase

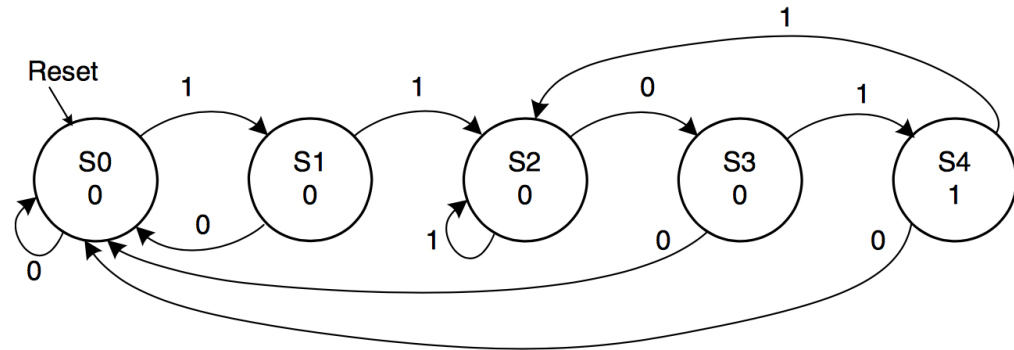
    assign q = (state == S0); // output logic
endmodule
```

FSM Example 2: Smiling Snail

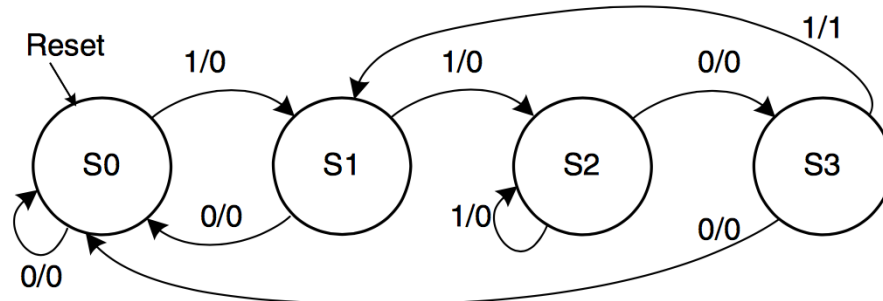
- Alyssa P. Hacker has a snail that crawls down a paper tape with 1's and 0's on it
- The snail smiles whenever the last four digits it has crawled over are **1101**
- Design Moore and Mealy FSMs of the snail's brain



Moore



Mealy



Implementing FSM Example 2: Definitions

```
module SmilingSnail (input clk,  
                    input reset,  
                    input number,  
                    output smile);
```

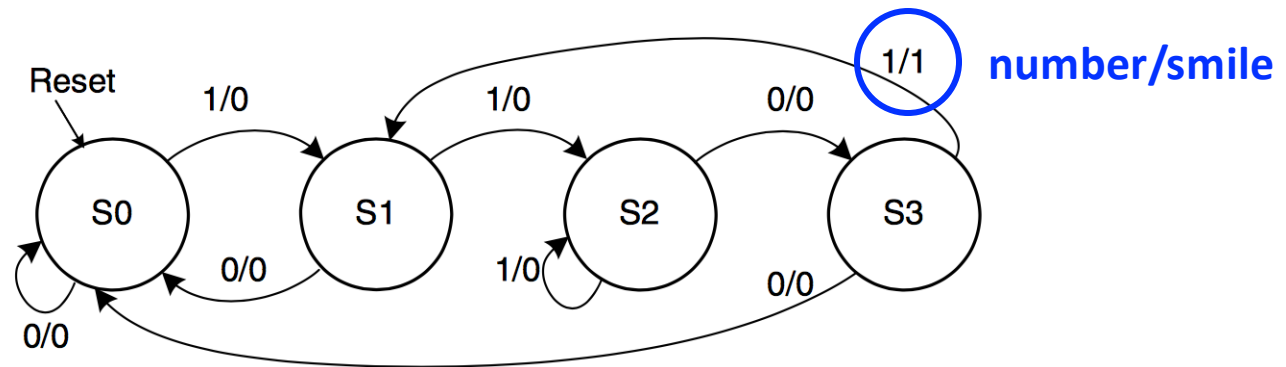
```
    reg [1:0] state, nextstate;
```

```
    parameter S0 = 2'b00;
```

```
    parameter S1 = 2'b01;
```

```
    parameter S2 = 2'b10;
```

```
    parameter S3 = 2'b11;
```



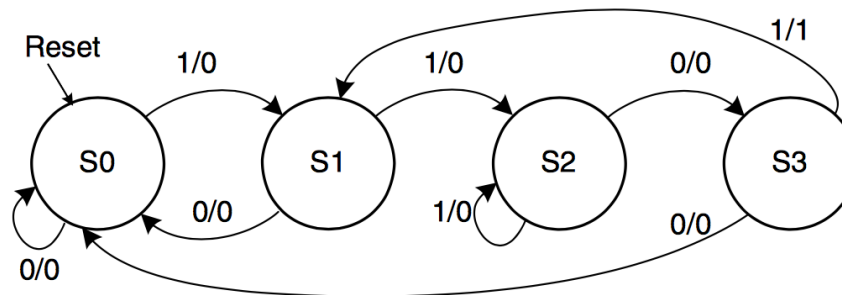
Implementing FSM Example 2: State Register

```
// state register
always @ (posedge clk, posedge reset)
    if (reset) state <= S0;
    else      state <= nextstate;
```

- This part defines the **state register** (memorizing process)
- Sensitive to only **clk**, **reset**
- In this example **reset** is active when '1' (active-high)

Implementing FSM Example 2: Next State Logic

```
// next state logic
always @ (*)
  case (state)
    S0: if (number) nextstate = S1;
        else nextstate = S0;
    S1: if (number) nextstate = S2;
        else nextstate = S0;
    S2: if (number) nextstate = S2;
        else nextstate = S3;
    S3: if (number) nextstate = S1;
        else nextstate = S0;
    default: nextstate = S0;
  endcase
```



Implementing FSM Example 2: Output Logic

```
// output logic
assign smile = (number & state == S3);
```

- In this example, output depends on state and input
 - **Mealy type FSM**
- We used a simple combinational assignment

Implementation of FSM Example 2

```
module SmilingSnail (input clk,
                    input reset,
                    input number,
                    output smile);

    reg [1:0] state, nextstate;

    parameter S0 = 2'b00;
    parameter S1 = 2'b01;
    parameter S2 = 2'b10;
    parameter S3 = 2'b11;

    // state register
    always @ (posedge clk, posedge
reset)
        if (reset) state <= S0;
        else      state <= nextstate;
```

```
    always @ (*) // next state logic
        case (state)
            S0: if (number)
                    nextstate = S1;
                else nextstate = S0;
            S1: if (number)
                    nextstate = S2;
                else nextstate = S0;
            S2: if (number)
                    nextstate = S2;
                else nextstate = S3;
            S3: if (number)
                    nextstate = S1;
                else nextstate = S0;
            default: nextstate = S0;
        endcase
    // output logic
    assign smile = (number & state==S3);

endmodule
```

What Did We Learn?

- Basics of describing **sequential circuits** in Verilog
- The **always** statement
 - ❑ Needed for defining memorizing elements (**flip-flops, latches**)
 - ❑ Can also be used to define **combinational circuits**
- **Blocking** vs **Non-blocking** statements
 - ❑ **=** assigns the value **immediately**
 - ❑ **<=** assigns the value **at the end of the block**
- **Describing FSMs in Verilog**
 - ❑ Next state logic
 - ❑ State assignment
 - ❑ Output logic

Next Lecture:

Timing and Verification

Digital Design & Computer Arch.

Lecture 7: Hardware Description Languages and Verilog

Prof. Onur Mutlu

ETH Zürich

Spring 2021

18 March 2021

Logic Simplification: Karnaugh Maps (K-Maps)

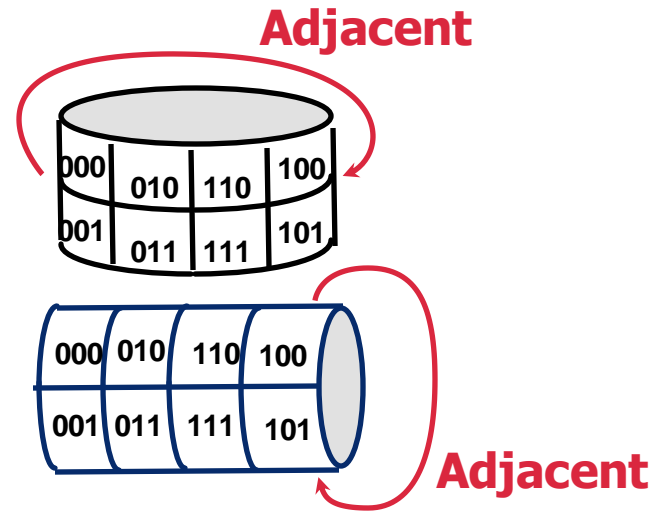
Karnaugh Maps are Fun...

- A pictorial way of minimizing circuits by visualizing opportunities for simplification
- They are for you to **study on your own...**

- See Backup Slides
- Read H&H Section 2.7
- Watch videos of Lectures 5 and 6 from 2019 DDCA course:
 - <https://youtu.be/0ks0PeaOUjE?list=PL5Q2soXY2Zi8J58xLKBNFQFHRO3GrXxA9&t=4570>
 - <https://youtu.be/ozs18ARNG6s?list=PL5Q2soXY2Zi8J58xLKBNFQFHRO3GrXxA9&t=220>

Karnaugh Map Methods

<i>BC</i> <i>A</i>	00	01	11	10
0	000	001	011	010
1	100	101	111	110



K-map adjacencies go "around the edges"
Wrap around from first to last column
Wrap around from top row to bottom row

Backup Slides on Karnaugh Maps (K-Maps)

Complex Cases

■ One example

$$Cout = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

■ Problem

- ❑ Easy to see how to apply Uniting Theorem...
- ❑ Hard to know if you applied it in all the right places...
- ❑ ...especially in a function of many more variables

■ Question

- ❑ Is there an easier way to find potential simplifications?
- ❑ i.e., potential applications of Uniting Theorem...?

■ Answer

- ❑ Need an intrinsically *geometric* representation for Boolean $f()$
- ❑ Something we can draw, see...

Karnaugh Map

- Karnaugh Map (K-map) method
 - K-map is an alternative method of representing the **truth table** that helps **visualize adjacencies** in up to 6 dimensions
 - Physical adjacency \leftrightarrow Logical adjacency

2-variable K-map

$A \backslash B$	0	1
0	00	01
1	10	11

3-variable K-map

$A \backslash BC$	00	01	11	10
0	000	001	011	010
1	100	101	111	110

4-variable K-map

$AB \backslash CD$	00	01	11	10
00	0000	0001	0011	0010
01	0100	0101	0111	0110
11	1100	1101	1111	1110
10	1000	1001	1011	1010

Numbering Scheme: 00, 01, 11, 10 is called a “Gray Code” — only a *single bit (variable) changes* from one code word and the next code word

K-map Cover - 4 Input Variables

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	1	1

$$F(A, B, C, D) = \sum m(0, 2, 5, 8, 9, 10, 11, 12, 13, 14, 15)$$

$$F = A + \bar{B}\bar{D} + B\bar{C}D$$

Strategy for “circling” rectangles on Kmap:

Biggest “oops!” that people forget:

Logic Minimization Using K-Maps

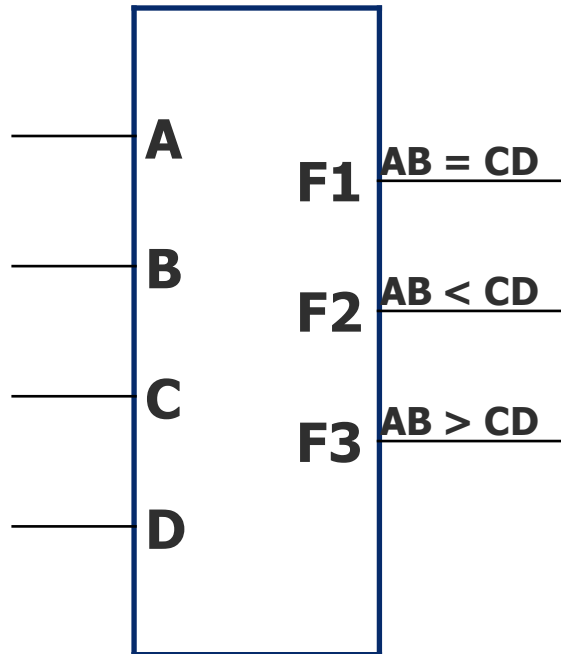
- Very simple guideline:
 - Circle all the rectangular blocks of 1's in the map, using the fewest possible number of circles
 - Each circle should be as large as possible
 - Read off the implicants that were circled

- More formally:
 - A Boolean equation is minimized when it is written as a sum of the fewest number of prime implicants
 - Each circle on the K-map represents an implicant
 - The largest possible circles are prime implicants

K-map Rules

- **What can be legally combined (circled) in the K-map?**
 - Rectangular groups of size 2^k for any integer k
 - Each cell has the same value (1, for now)
 - All values must be adjacent
 - Wrap-around edge is okay
- **How does a group become a term in an expression?**
 - Determine which literals are constant, and which vary across group
 - Eliminate varying literals, then AND the constant literals
 - constant 1 → use X , constant 0 → use \bar{X}
- **What is a good solution?**
 - Biggest groupings → eliminate more variables (literals) in each term
 - Fewest groupings → fewer terms (gates) all together
 - OR together all AND terms you create from individual groups

K-map Example: Two-bit Comparator



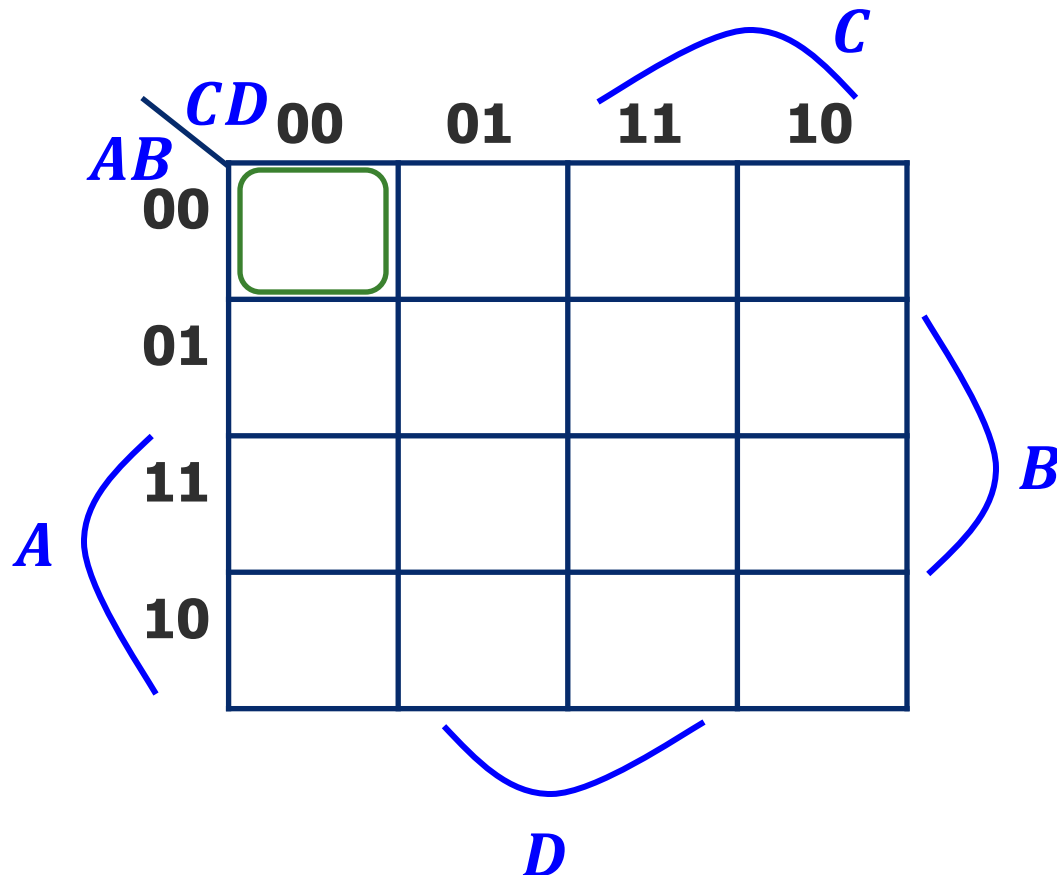
Design Approach:

Write a 4-Variable K-map
for each of the 3
output functions

A	B	C	D	F1	F2	F3
0	0	0	0	1	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	1	0	0
0	1	1	0	0	1	0
0	1	1	1	0	1	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	1	0	0
1	0	1	1	0	1	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	1	0	0

K-map Example: Two-bit Comparator (2)

K-map for F1

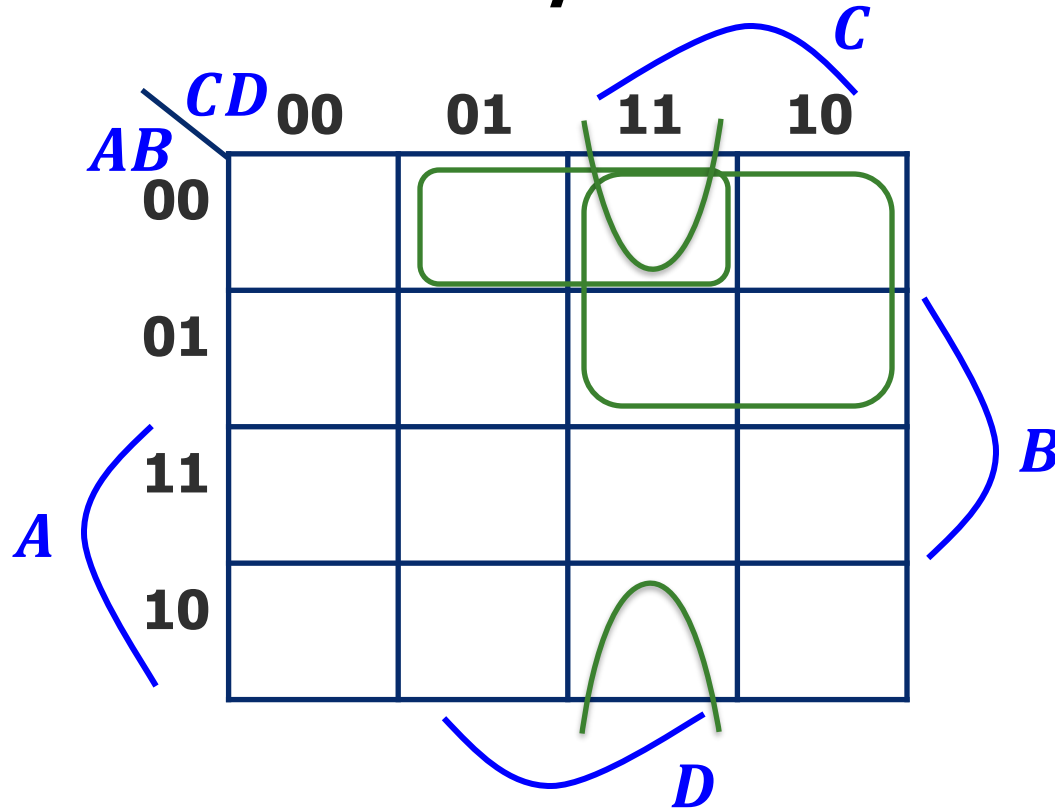


F1 =

A	B	C	D	F1	F2	F3
0	0	0	0	1	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	1	0	0
0	1	1	0	0	1	0
0	1	1	1	0	1	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	1	0	0
1	0	1	1	0	1	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	1	0	0

K-map Example: Two-bit Comparator (3)

K-map for F2



F2 =

F3 = ? (Exercise for you)

A	B	C	D	F1	F2	F3
0	0	0	0	1	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	1	0	0
0	1	1	0	0	1	0
0	1	1	1	0	1	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	1	0	0
1	0	1	1	0	1	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	1	0	0

K-maps with “Don’t Care”

- **Don’t Care** really means *I don’t care what my circuit outputs if this appears as input*
 - You have an engineering choice to use DON’T CARE patterns intelligently as 1 or 0 to better **simplify** the circuit

A	B	C	D	F	G
...					
0	1	1	0	X	X
0	1	1	1		
1	0	0	0	X	X
1	0	0	1		
...					

I can pick 00, 01, 10, 11 independently of below

I can pick 00, 01, 10, 11 independently of above

Example: BCD Increment Function

- BCD (Binary Coded Decimal) digits
 - Encode decimal digits 0 - 9 with bit patterns 0000_2 — 1001_2
 - When **incremented**, the decimal sequence is 0, 1, ..., 8, 9, 0, 1

A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

These input patterns **should never be encountered** in practice
(hey -- it's a BCD number!)
So, associated output values are
"Don't Cares"

K-map for BCD Increment Function

A B

+

W X

Z (without don't cares) =

Z (with don't cares) =

10	1		X	X
-----------	----------	--	----------	----------

10			X	X
-----------	--	--	----------	----------

Y

		CD			
		00	01	11	10
AB	00		1		1
	01		1		1
	11	X	X	X	X
	10			X	X

Z

		CD			
		00	01	11	10
AB	00	1			1
	01	1			1
	11	X	X	X	X
	10	1		X	X

A **B** **C** **D**

K-map Summary

- Karnaugh maps as a formal systematic approach for logic simplification
- 2-, 3-, 4-variable K-maps
- K-maps with “Don’t Care” outputs
- H&H Section 2.7