Design of Digital Circuits

Lab 1 Supplement: Drawing Basic Circuits

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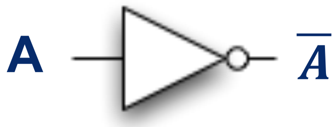
What We Will Learn?

- In Lab 1, you will design simple combinatorial circuits
- We will cover a tutorial about:
  - Boolean Equations
    - Logic operations with binary numbers
  - Logic Gates
    - Basic blocks that are interconnected to form larger units that are needed to construct a computer

Boolean Equations and Logic Gates

Simple Equations: NOT / AND / OR

$\overline{A}$ *(reads "not A")* is 1 iff A is 0



A	$\overline{A}$
0	1
1	0

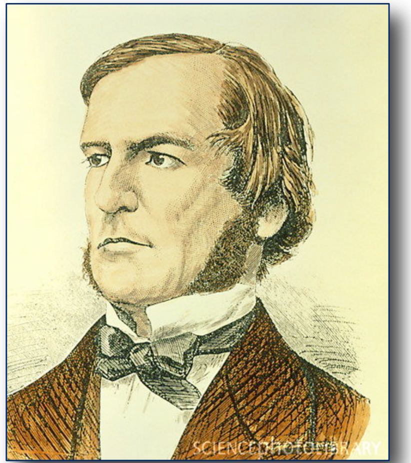
A • B *(reads "A and B")* is 1 iff A and B are both 1

A	B	$A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

A + B *(reads "A or B")* is 1 iff either A or B is 1

A	B	$A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Algebra: Big Picture



- An algebra on 1's and 0's
  - with AND, OR, NOT operations
- What you start with
  - Axioms: basic stuff about objects and operations you just assume to be true at the start
- What you derive first
  - Laws and theorems: allow you to manipulate Boolean expressions
  - …also allow us to do some simplification on Boolean expressions
- What you derive later
  - More "sophisticated" properties useful for manipulating digital designs represented in the form of Boolean equations

Common Logic Gates

Buffer

A	Z
0	0
1	1

AND

A	B	Z
0	0	0
0	1	0
1	0	0
1	1	1

OR

A	B	Z
0	0	0
0	1	1
1	0	1
1	1	1

XOR

A	B	Z
0	0	0
0	1	1
1	0	1
1	1	0

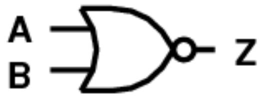
Inverter

A	Z
0	1
1	0

NAND

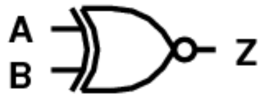
A	B	Z
0	0	1
0	1	1
1	0	1
1	1	0

NOR



A	B	Z
0	0	1
0	1	0
1	0	0
1	1	0

XNOR



A	B	Z
0	0	1
0	1	0
1	0	0
1	1	1

Boolean Algebra: Axioms

Formal version	*English version*
1. B contains at least two elements, 0 and 1, *such that* $0 \neq 1$	Math formality...
2. *Closure* $a,b \in B$, (i) $a + b \in B$ (ii) $a \bullet b \in B$	Result of AND, OR stays in set you start with
3. *Commutative Laws*: $a,b \in B$, (i) $a + b = b + a$ (ii) $a \bullet b = b \bullet a$	For primitive AND, OR of 2 inputs, order doesn't matter
4. *Identities*: $0, 1 \in B$ (i) $a + 0 = a$ (ii) $a \bullet 1 = a$	There are identity elements for AND, OR, give you back what you started with
5. *Distributive Laws*: (i) $a + (b \bullet c) = (a + b) \bullet (a + c)$ (ii) $a \bullet (b + c) = a \bullet b + a \bullet c$	• distributes over +, just like algebra …but + distributes over •, also (!!)
6. *Complement*: (i) $a + a' = 1$ (ii) $a \bullet a' = 0$	There is a complement element, ANDing, ORing give you an identity

Boolean Algebra: Duality

- Interesting observation
  - All the axioms come in "dual" form
  - Anything true for an expression also true for its dual
  - So any derivation you could make that is true, can be flipped into dual form, and it stays true

- Duality -- More formally
  - A dual of a Boolean expression is derived by replacing
    - Every AND operation with... an OR operation
    - Every OR operation with... an AND
    - Every constant 1 with... a constant 0
    - Every constant 0 with... a constant 1
    - But don't change any of the literals or play with the complements!

Example

$$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$$

$$\rightarrow \; a + (b \bullet c) = (a + b) \bullet (a + c)$$

Boolean Algebra: Useful Laws

Dual

Operations with 0 and 1:

1. $X + 0 = X$ — 1D. $X \bullet 1 = X$
2. $X + 1 = 1$ — 2D. $X \bullet 0 = 0$

AND, OR with identities gives you back the original variable or the identity

Idempotent Law:

3. $X + X = X$ — 3D. $X \bullet X = X$

AND, OR with self = self

Involution Law:

4. $\overline{(\overline{X})} = X$

double complement = no complement

Laws of Complementarity:

5. $X + \overline{X} = 1$ — 5D. $X \bullet \overline{X} = 0$

AND, OR with complement gives you an identity

Commutative Law:

6. $X + Y = Y + X$ — 6D. $X \bullet Y = Y \bullet X$

Just an axiom…

Useful Laws (cont.)

Associative Laws:

7. $(X + Y) + Z = X + (Y + Z)$
$= X + Y + Z$

7D. $(X \bullet Y) \bullet Z = X \bullet (Y \bullet Z)$
$= X \bullet Y \bullet Z$

Parenthesis order doesn't matter

Distributive Laws:

8. $X \bullet (Y + Z) = (X \bullet Y) + (X \bullet Z)$

8D. $X + (Y \bullet Z) = (X + Y) \bullet (X + Z)$

Axiom

Simplification Theorems:

9. $X \bullet Y + X \bullet \overline{Y} = X$

9D. $(X + Y) \bullet (X + \overline{Y}) = X$

10. $X + X \bullet Y = X$

10D. $X \bullet (X + Y) = X$

11. $(X + \overline{Y}) \bullet Y = X \bullet Y$

11D. $(X \bullet \overline{Y}) + Y = X + Y$

Useful for simplifying expressions

Actually worth remembering — they show up a lot in real designs…

DeMorgan's Law

DeMorgan's Law:

12. $\overline{(X + Y + Z + \cdots)} = \overline{X}.\overline{Y}.\overline{Z}. \ldots$

12D. $\overline{(X . Y . Z. \ldots)} = \overline{X} + \overline{Y} + \overline{Z} + \ldots$

- **Think of this as a transformation**
  - Let's say we have:

$$F = A + B + C$$

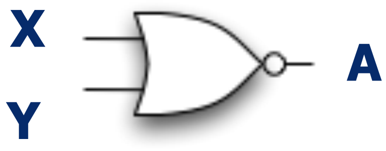
  - Applying DeMorgan's Law (12), gives us:

$$F = \overline{\overline{(A + B + C)}} = \overline{(\overline{A}.\overline{B}.\overline{C})}$$

DeMorgan's Law (cont.)

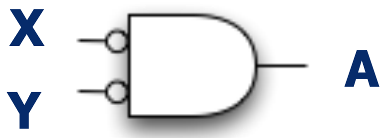
Interesting — these are conversions between different types of logic
That's useful given you don't always have every type of gate

$$A = \overline{(X+Y)} = \overline{X}\,\overline{Y}$$



X	Y	$\overline{X+Y}$	$\overline{X}$	$\overline{Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

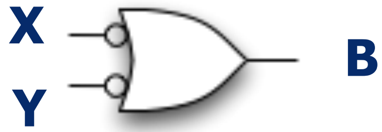
NOR is equivalent to AND with inputs complemented



$$B = \overline{(XY)} = \overline{X} + \overline{Y}$$

X	Y	$\overline{XY}$	$\overline{X}$	$\overline{Y}$	$\overline{X}+\overline{Y}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

NAND is equivalent to OR with inputs complemented



Part 1: A Comparator Circuit

- Design a comparator that receives two 4-bit numbers A and B, and sets the output bit EQ to logic-1 if A and B are equal

- Hints:
  - First compare A and B bit by bit
  - Then combine the results of the previous steps to set EQ to logic-1 if all A and B are equal

Part 2: A More General Comparator

- Design a circuit that receives two 1-bit inputs A and B, and:
  - sets its first output (O1) to 1 if *A>B*,
  - sets the second output (O2) to 1 if *A=B*,
  - sets the third output (O3) to 1 if *A<B*.

Part 3: Circuits with Only NAND Gates

- Design the circuit of Part 2 using only NAND gates

- Logical Completeness:
  - The set of gates {AND, OR, NOT} is logically complete because we can build a circuit to carry out the specification of any combinatorial logic we wish, without any other kind of gate
  - NAND and NOR are also logically complete

Last Words

- In this lab, you will draw the schematics of some simple operations
- Part 1: A comparator circuit
- Part 2: A more general comparator circuit
- Part 3: Designing circuits using only NAND gates
- You will find more exercises in the lab report

Report Deadline

23:59, 20 March 2020

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