P&S Heterogeneous Systems

Parallel Patterns: Prefix Sum (Scan)

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Parallel Patterns

Reduction Operation

- A reduction operation reduces a set of values to a single value
  - Sum, Product, Minimum, Maximum are examples

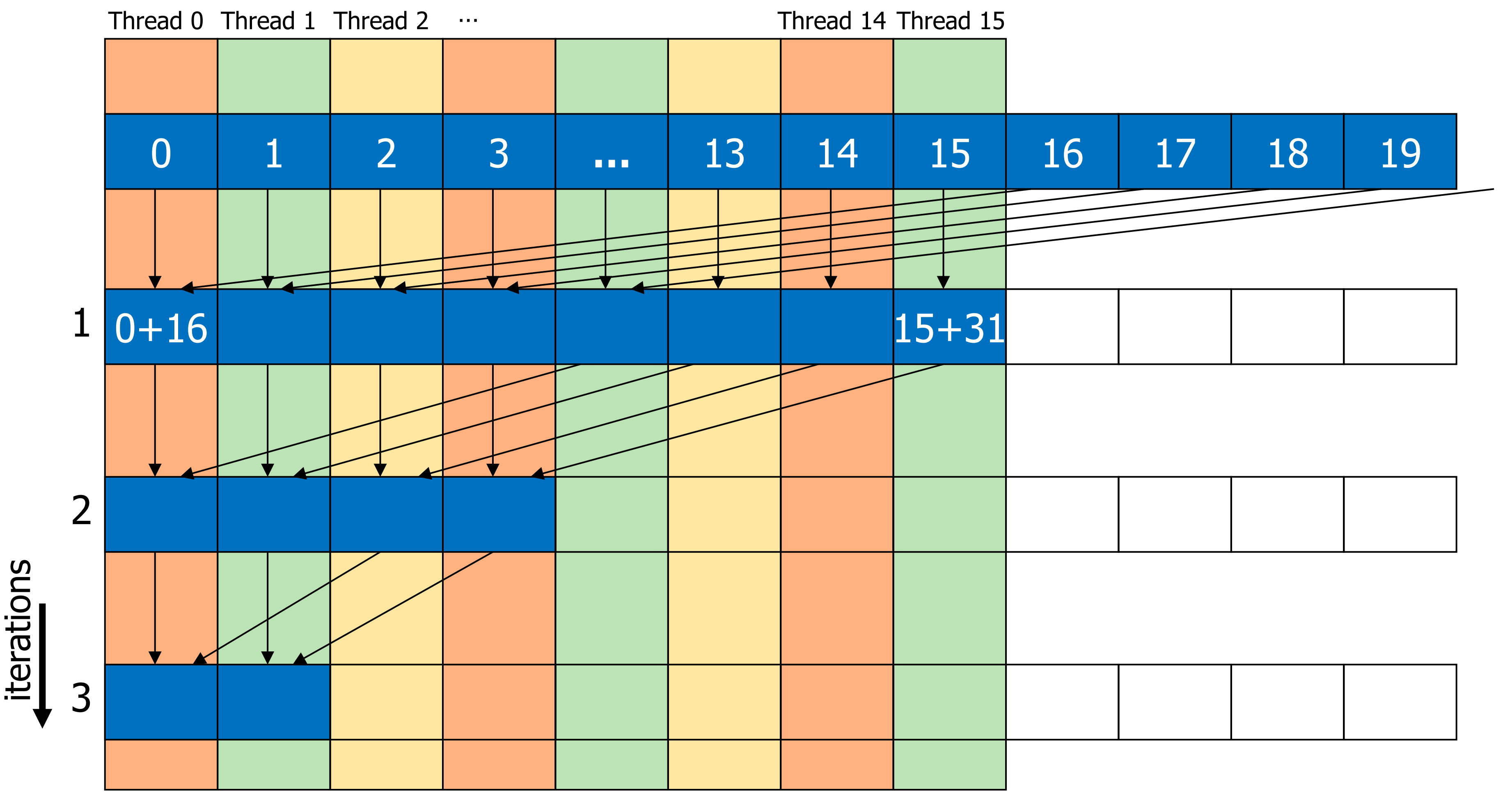
- Properties of reduction
  - Associativity
  - Commutativity
  - Identity value

- Reduction is a key primitive for parallel computing
  - E.g., MapReduce programming model

Dean and Ghemawat, "MapReduce: Simplified Data Processing of Large Clusters," OSDI 2004

Divergence-Free Mapping (I)

- All active threads belong to the same warp



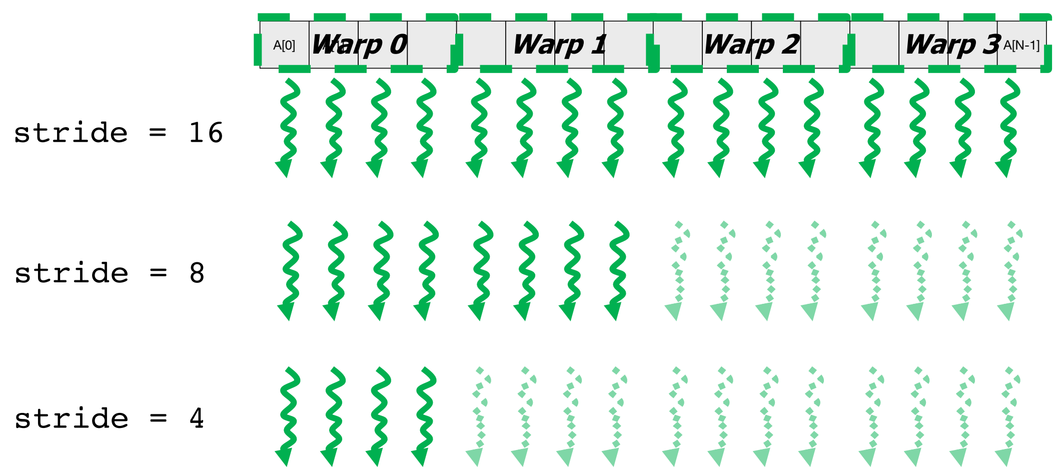
Slide credit: Hwu & Kirk

Divergence-Free Mapping (II)

- Program with high SIMD utilization

```
__shared__ float partialSum[]

unsigned int t = threadIdx.x;

for(int stride = blockDim.x; stride > 0;  stride >> 1){

  __syncthreads();

  if (t < stride)
    partialSum[t] += partialSum[t + stride];

}
```

Warp utilization
is maximized



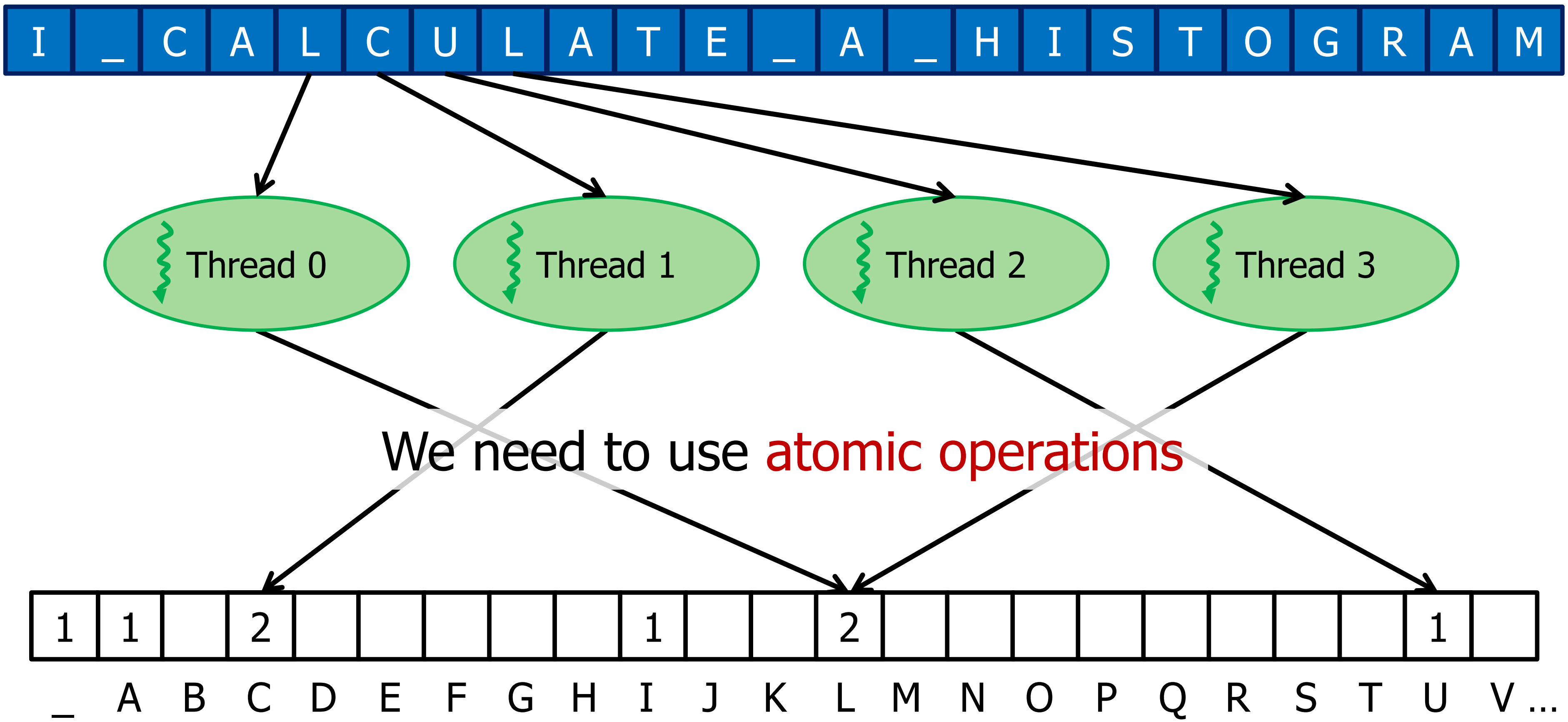
Histogram Computation

- Histogram is a frequently used computation for reducing the dimensionality and extracting notable features and patterns from large data sets
  - Feature extraction for object recognition in images
  - Fraud detection in credit card transactions
  - Correlating heavenly object movements in astrophysics
  - …
- Basic histograms - for each element in the data set, use the value to identify a “bin” to increment
  - Divide possible input value range into “bins”
  - Associate a counter to each bin
  - For each input element, examine its value and determine the bin it falls into and increment the counter for that bin

Slide credit: Hwu & Kirk

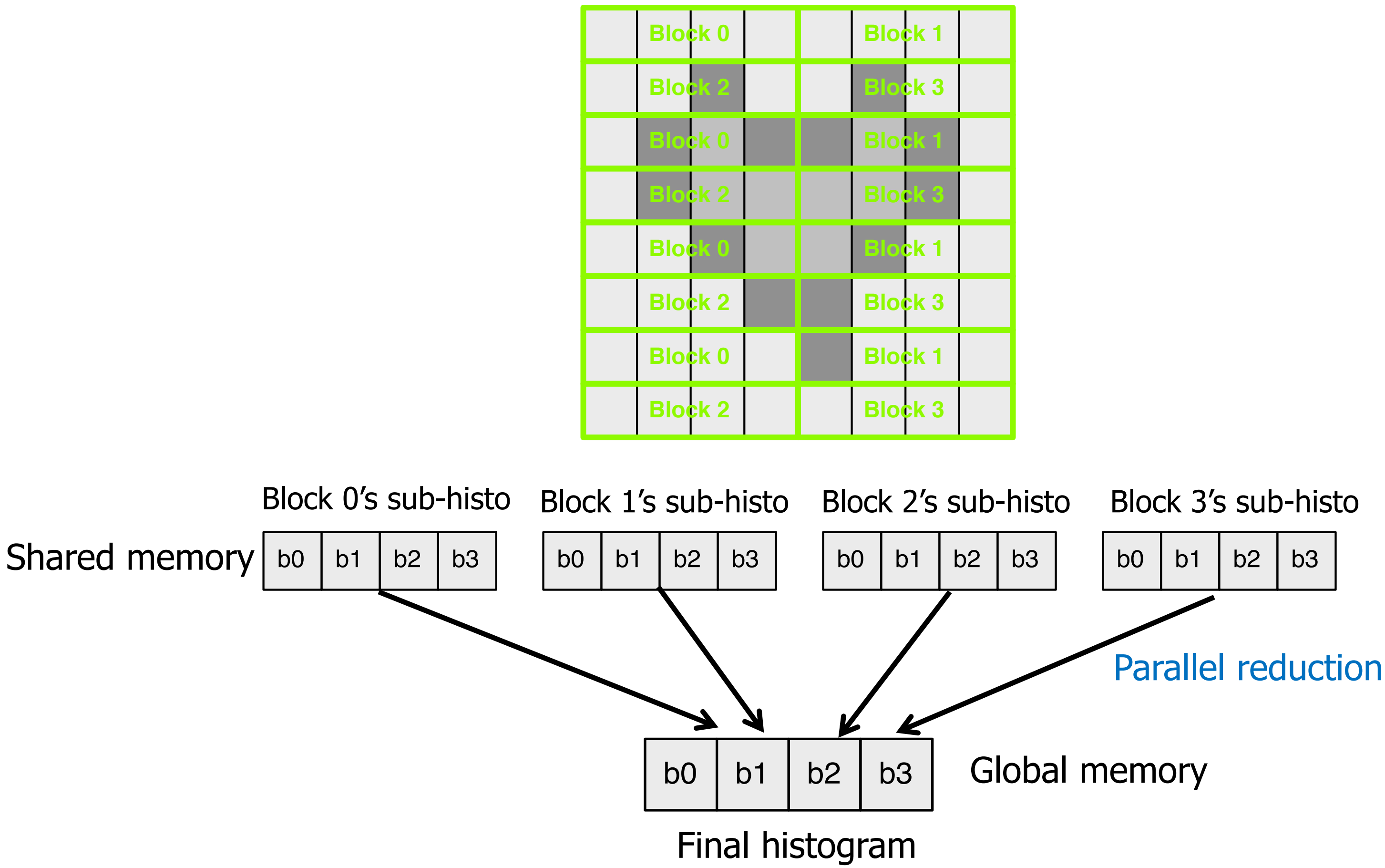
Parallel Histogram Computation: Iteration 2

- All threads move to the next section of the input
  - Each thread moves to element threadID + #threads



Histogram Privatization

- Privatization: Per-block sub-histograms in shared memory
  - Threads use atomic operations in shared memory



Convolution Applications

- Convolution is a widely-used operation in signal processing, image processing, video processing, and computer vision

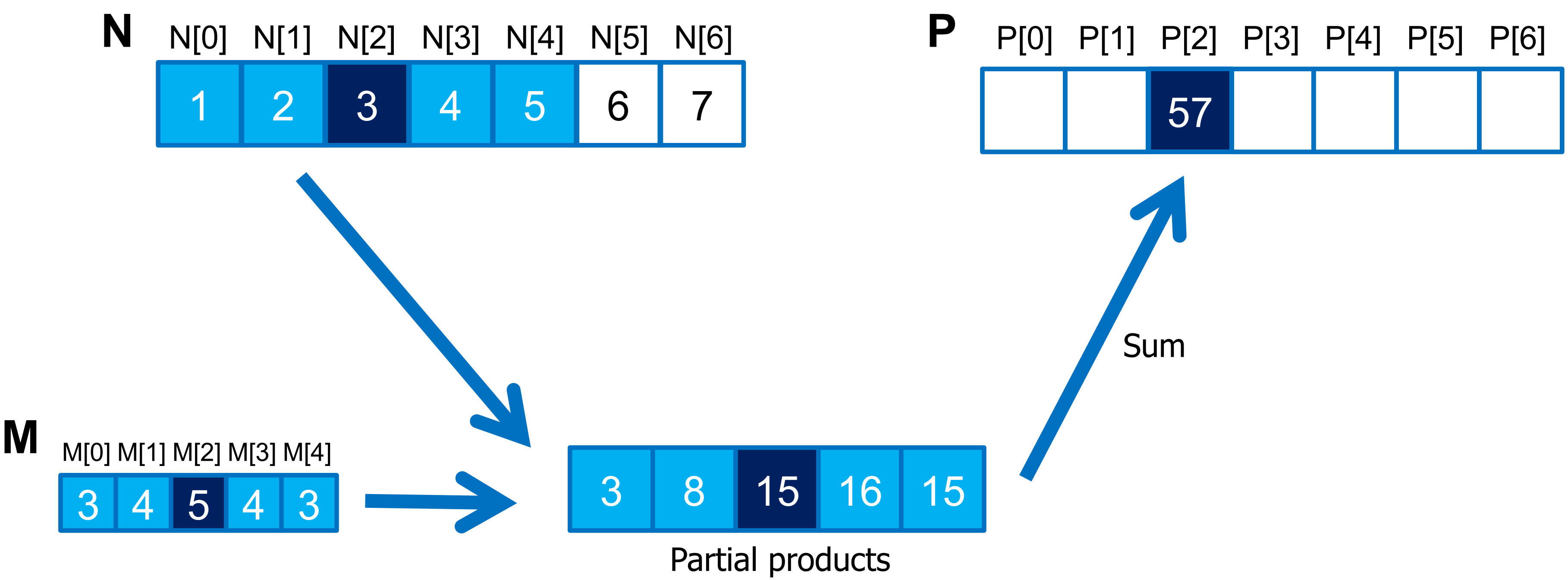
- Convolution applies a filter or mask or kernel* on each element of the input (e.g., a signal, an image, a frame) to obtain a new value, which is a weighted sum of a set of neighboring input elements
  - Smoothing, sharpening, or blurring an image
  - Finding edges in an image
  - Removing noise, etc.

- Applications in machine learning and artificial intelligence
  - Convolutional Neural Networks (CNN or ConvNets)

* The term "kernel" may create confusion in the context of GPUs (recall a CUDA/GPU kernel is a function executed by a GPU)

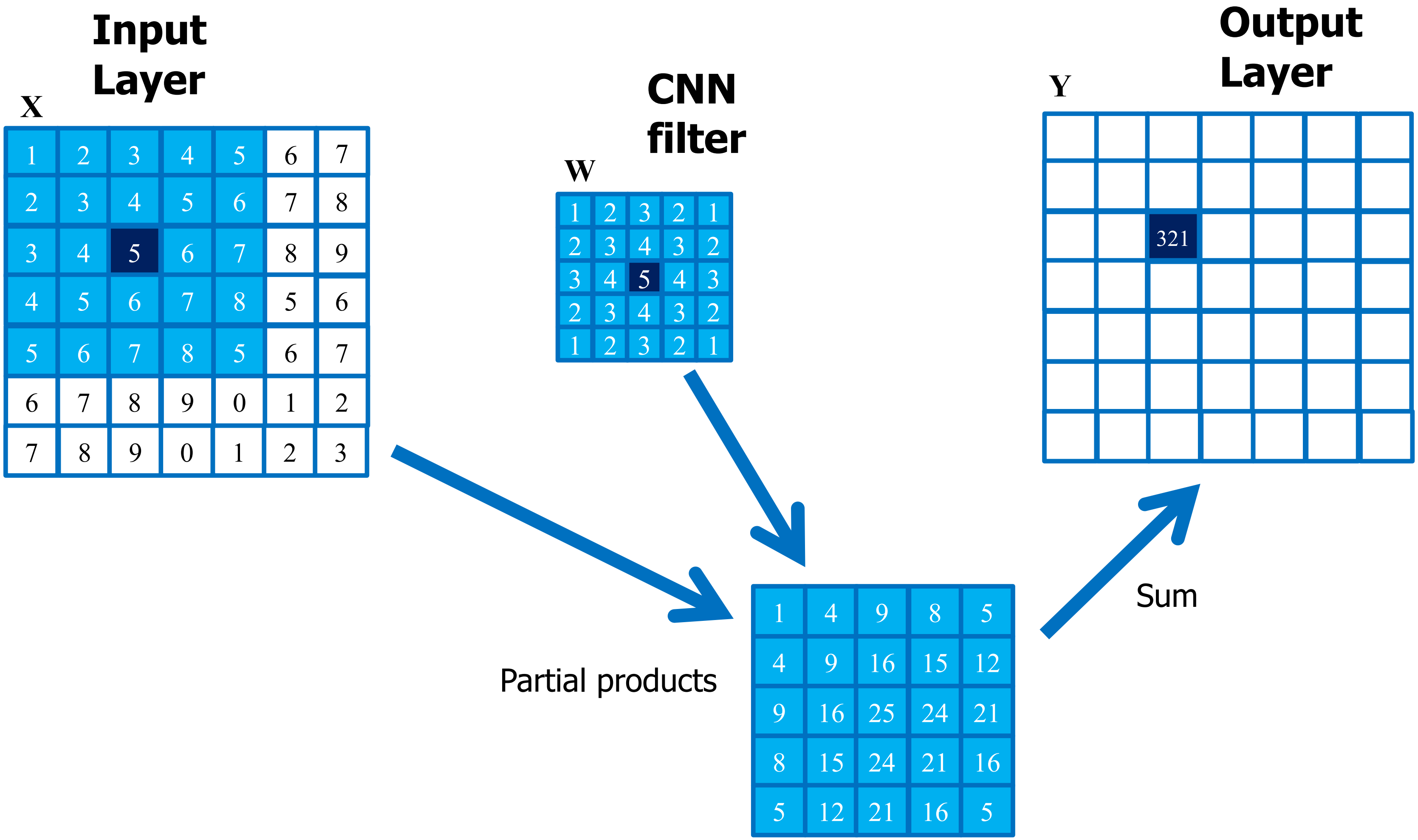
1D Convolution Example

- Commonly used for audio processing
- Mask size is usually an odd number of elements for symmetry (5 in this example)
- Calculation of P[2]:

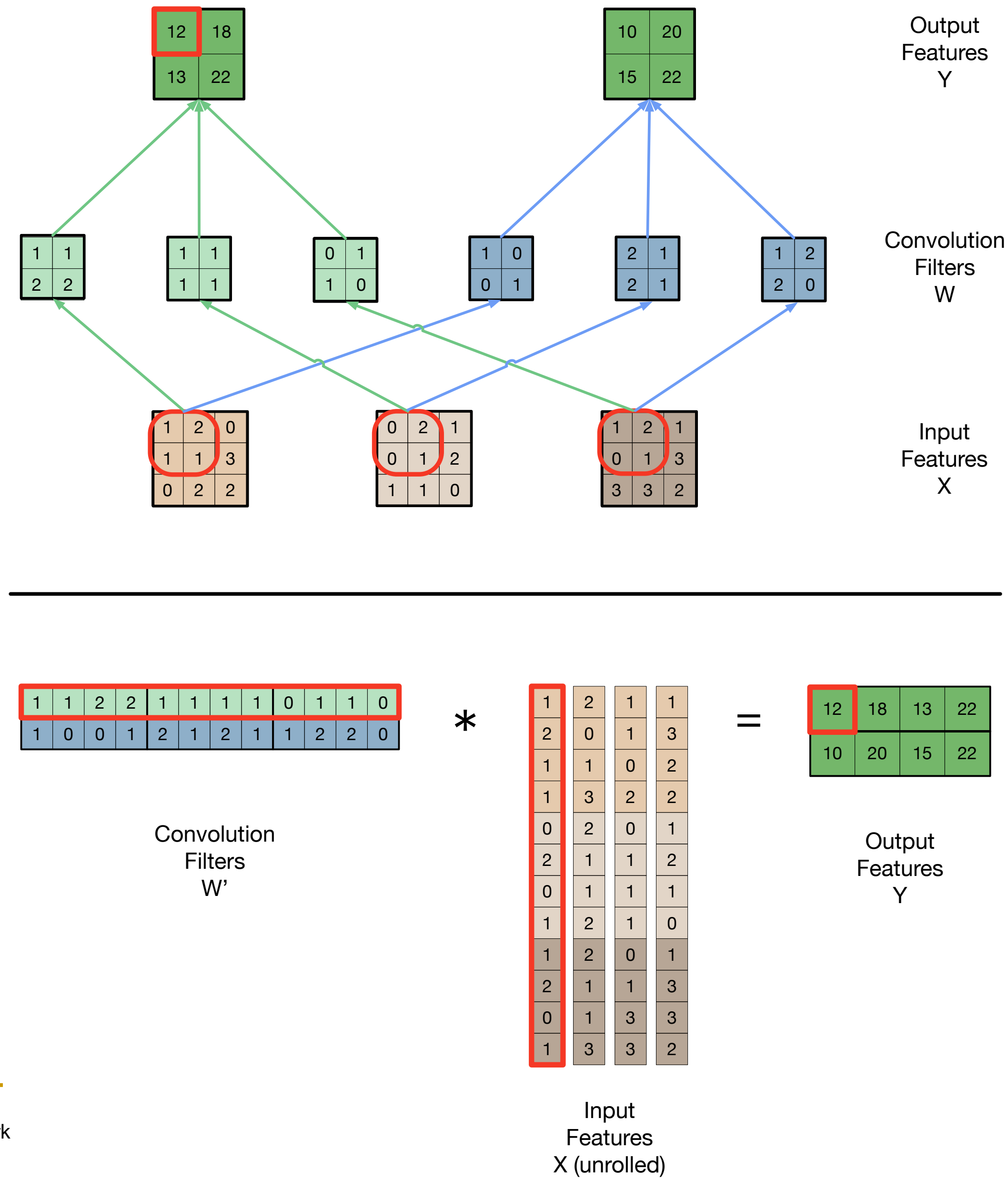


Slide credit: Hwu & Kirk

Another Example of 2D Convolution



Implementing a Convolutional Layer with Matrix Multiplication



Slide credit: Reproduced from Hwu & Kirk

Prefix Sum (Scan)

Prefix Sum (Scan)

- Prefix sum or scan is an operation that takes an input array and an associative operator,
  - E.g., addition, multiplication, maximum, minimum
- And returns an output array that is the result of recursively applying the associative operator on the elements of the input array

- Input array $[x_0, x_1, \ldots, x_{n-1}]$
- Associative operator $\oplus$

- An output array $[y_0, y_1, \ldots, y_{n-1}]$ where
  - Exclusive scan: $y_i = x_0 \oplus x_1 \oplus \ldots \oplus x_{i-1}$
  - Inclusive scan: $y_i = x_0 \oplus x_1 \oplus \ldots \oplus x_i$

Scan Applications

- Scan is a key parallel primitive that can
  - convert recurrences from sequential

```
for(int i=1; i<n; i++)
    out[i] = out[i-1] + f(i);
```

  - into parallel

```
forall(i) {temp[i] = f(i)};
scan(out, temp);
```

- Scan is a basic building block of many parallel algorithms
  - E.g., stream compaction, partition, select, unique, radix sort, quicksort, string comparison, lexical analysis, polynomial evaluation, solving recurrences, tree operations, histograms, etc.

Examples of Exclusive and Inclusive Scan

Input

1	2	3	4	1	1	1	1	0	1	2	3	2	2	2	2

Output (Exclusive Scan)

```
out[0] = 0; // Identity value
for(int i=1; i<n; i++)
    out[i] = out[i-1] + in[i-1];
```

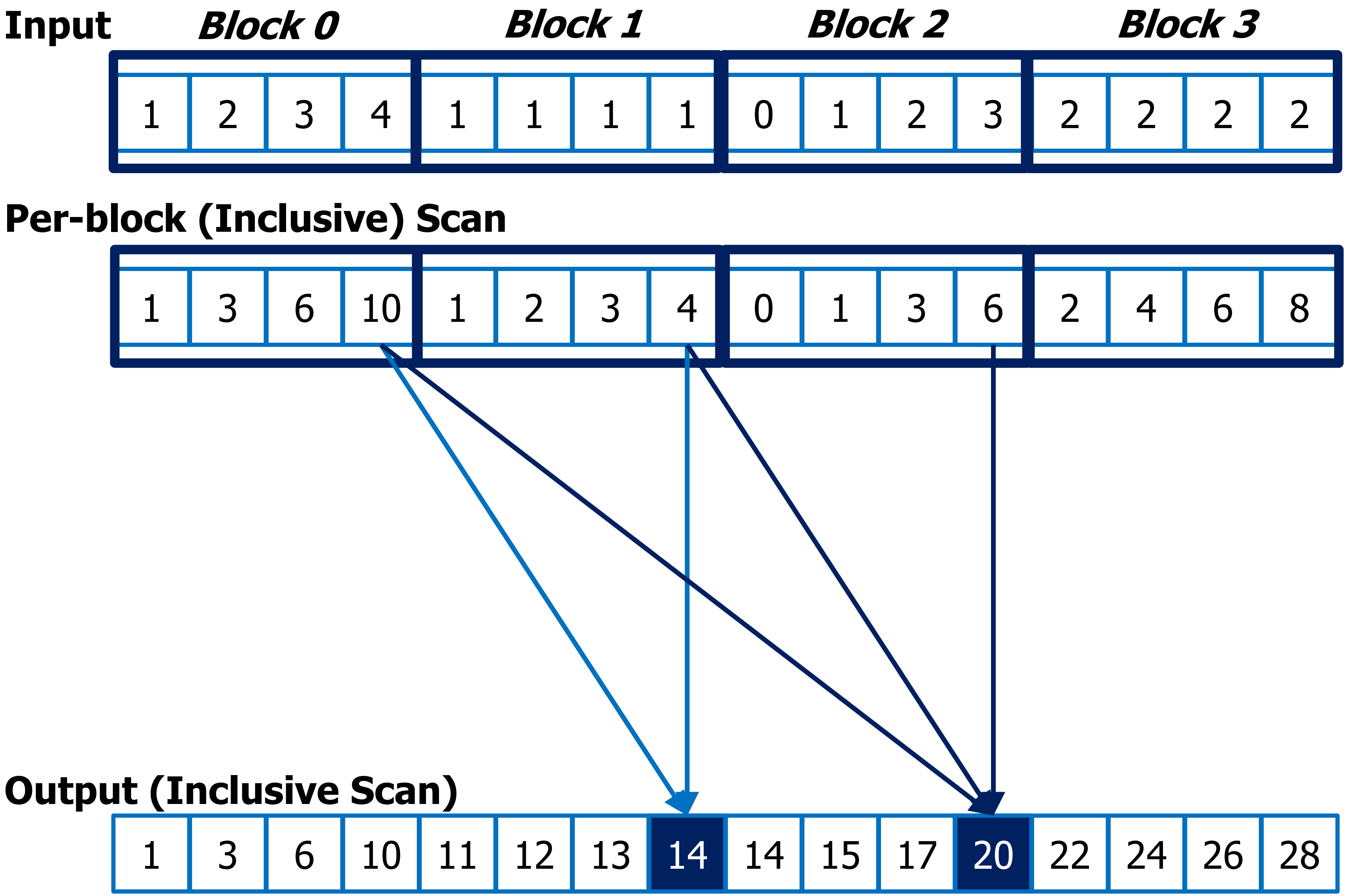
0	1	3	6	10	11	12	13	14	14	15	17	20	22	24	26

Output (Inclusive Scan)

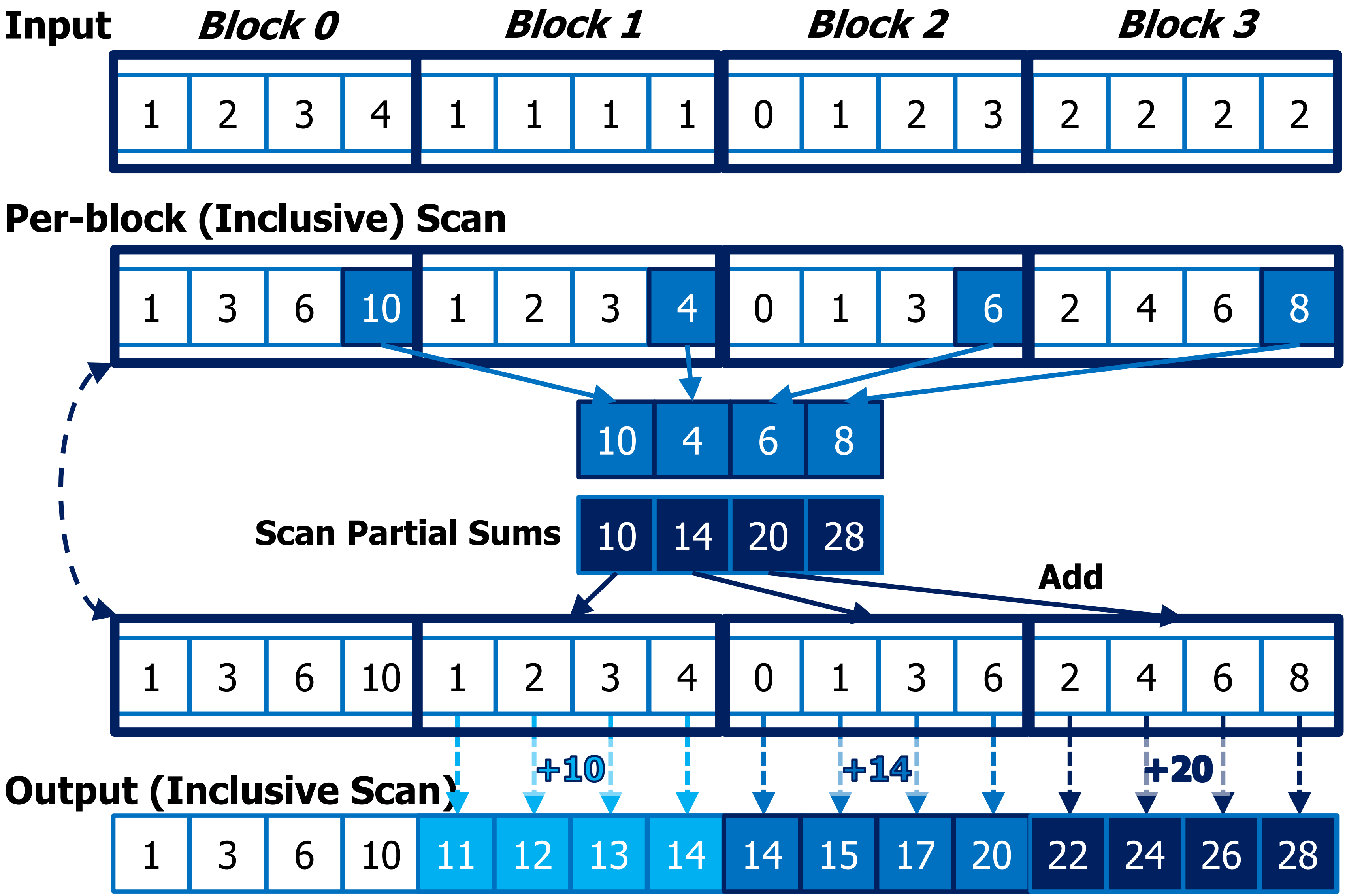
```
out[0] = in[0];
for(int i=1; i<n; i++)
    out[i] = out[i-1] + in[i];
```

1	3	6	10	11	12	13	14	14	15	17	20	22	24	26	28

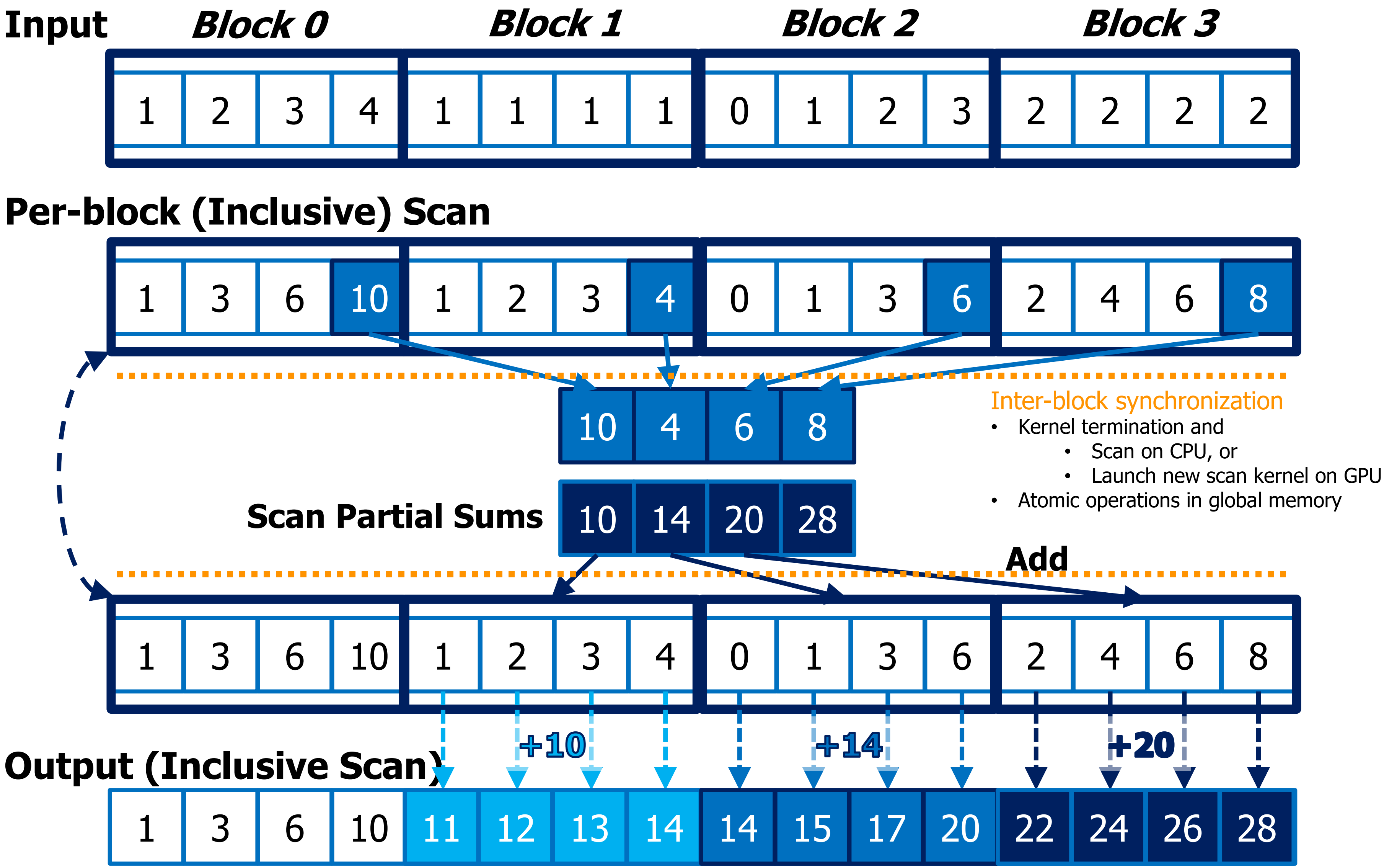
Hierarchical (Inclusive) Scan



Hierarchical (Inclusive) Scan



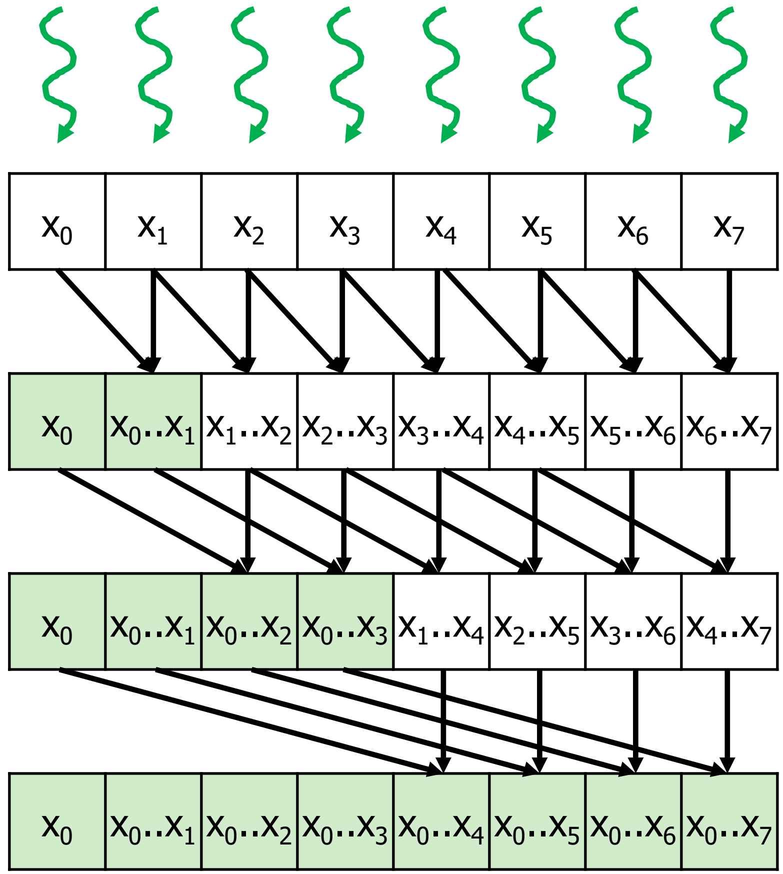
Hierarchical (Inclusive) Scan



Per-Block (Inclusive) Scan

- Inside a thread block, we can also apply a hierarchical approach
  - Warps
  - Threads
- Let’s start with the basic algorithms
  - Kogge-Stone
  - Brent-Kung

Kogge-Stone Parallel (Inclusive) Scan

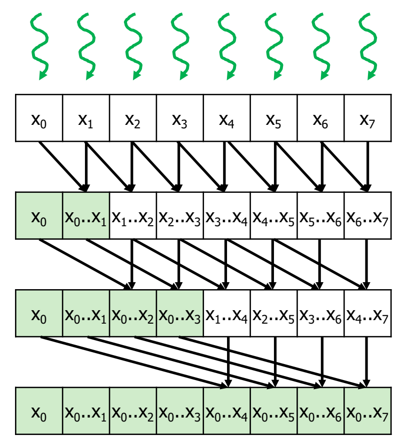


Slide credit: Izzat El Hajj

Kogge-Stone Parallel (Inclusive) Scan Code

```
unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;

output[i] = input[i];

__syncthreads();

for(unsigned int stride = 1; stride <= BLOCK_DIM/2; stride *= 2) {
    float v;
    if(threadIdx.x >= stride) {
        v = output[i - stride];
    }
    __syncthreads();

    if(threadIdx.x >= stride) {
        output[i] += v;
    }
    __syncthreads();
}

if(threadIdx.x == BLOCK_DIM - 1) {
    partialSums[blockIdx.x] = output[i];
}
```

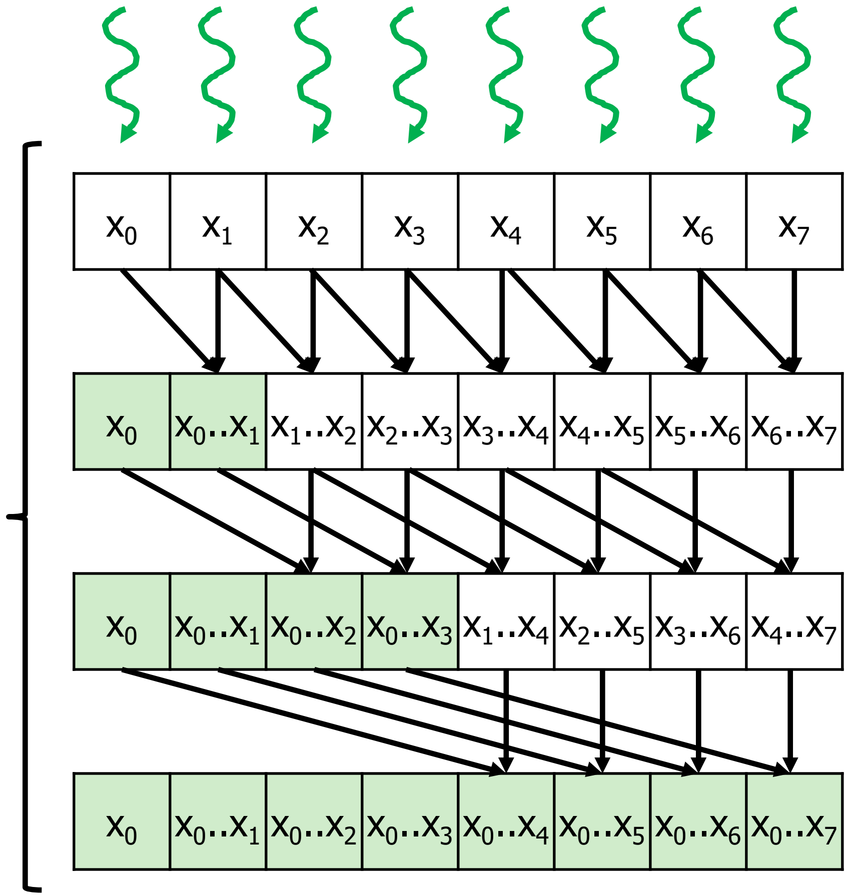
Wait for everyone to read before updating



Slide credit: Izzat El Hajj

Kogge-Stone Parallel (Inclusive) Scan

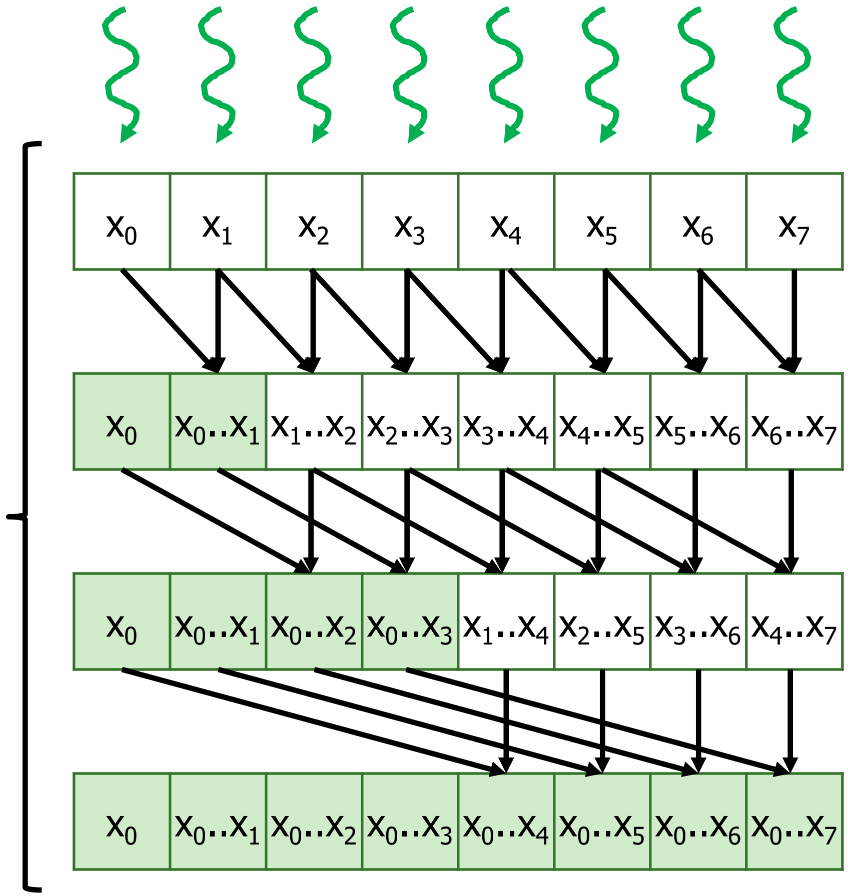
Observation:
memory locations are reused



Slide credit: Izzat El Hajj

Using Shared Memory

<u>Optimization:</u> load once to a **shared memory** buffer and perform successive reads and writes to the same array can be done in shared memory



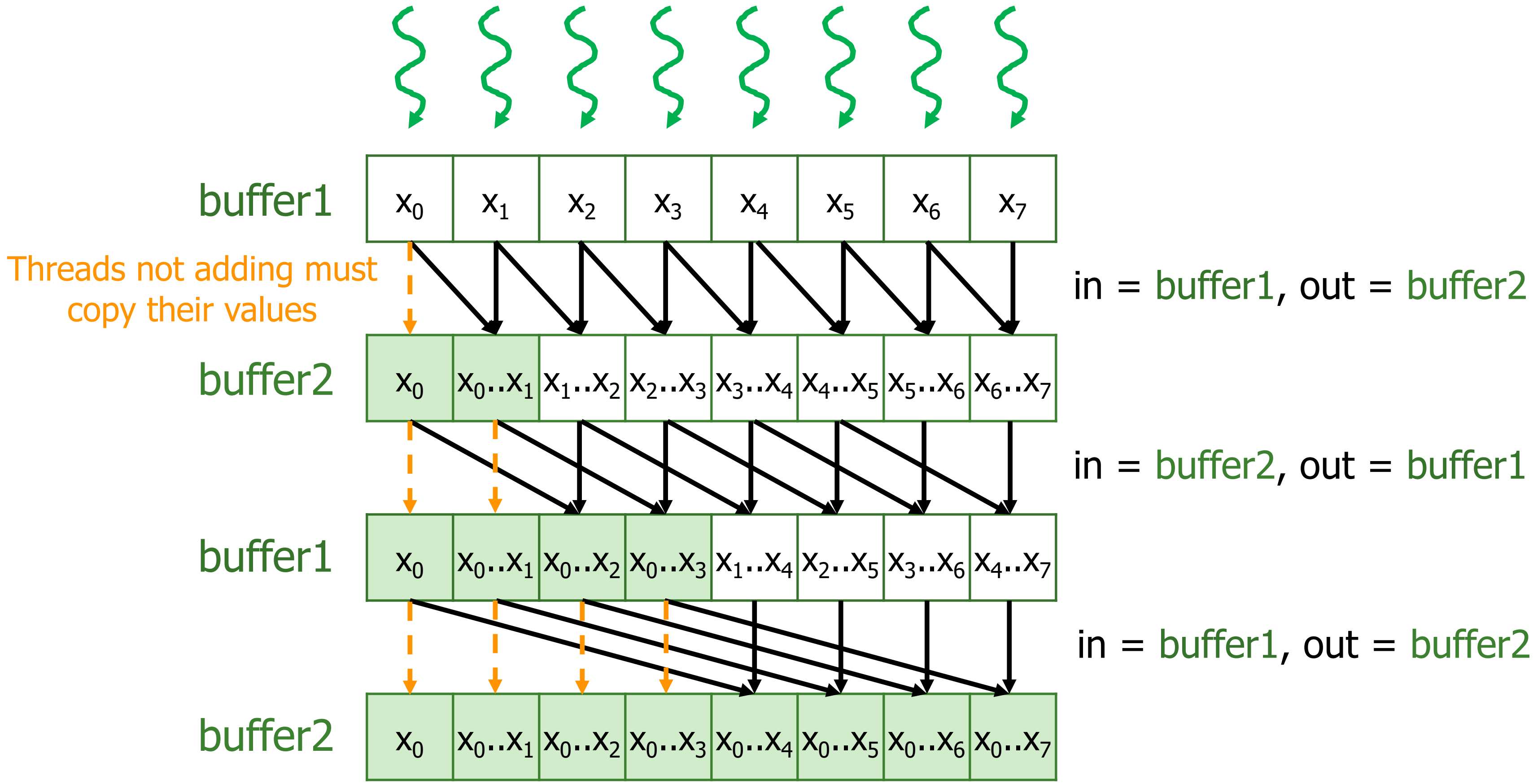
Slide credit: Izzat El Hajj

Using Shared Memory Code

```
unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;

__shared__ float buffer_s[BLOCK_DIM];
buffer_s[threadIdx.x] = input[i];
__syncthreads();

for(unsigned int stride = 1; stride <= BLOCK_DIM/2; stride *= 2) {
    float v;
    if(threadIdx.x >= stride) {
        v = buffer_s[threadIdx.x - stride];
    }
    __syncthreads();

    if(threadIdx.x >= stride) {
        buffer_s[threadIdx.x] += v;
    }
    __syncthreads();
}

if(threadIdx.x == BLOCK_DIM - 1) {
    partialSums[blockIdx.x] = buffer_s[threadIdx.x];
}

output[i] = buffer_s[threadIdx.x];
```


Slide credit: Izzat El Hajj

Double Buffering



Optimization: eliminate one synchronization by using two buffers and alternating them as the input/output buffer (called **double buffering**)

Slide credit: Izzat El Hajj

Double Buffering

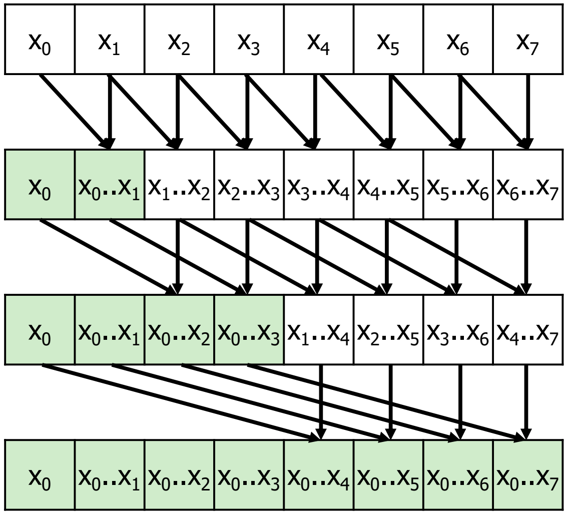
```
unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;

__shared__ float buffer1_s[BLOCK_DIM];
__shared__ float buffer2_s[BLOCK_DIM];
float* inBuffer_s = buffer1_s;
float* outBuffer_s = buffer2_s;
inBuffer_s[threadIdx.x] = input[i];
__syncthreads();

for(unsigned int stride = 1; stride <= BLOCK_DIM/2; stride *= 2) {
    if(threadIdx.x >= stride) {
        outBuffer_s[threadIdx.x] =
                inBuffer_s[threadIdx.x] + inBuffer_s[threadIdx.x - stride];
    }
    else {
        outBuffer_s[threadIdx.x] = inBuffer_s[threadIdx.x];
    }
    __syncthreads();

    float* tmp = inBuffer_s;
    inBuffer_s = outBuffer_s;
    outBuffer_s = tmp;
}

if(threadIdx.x == BLOCK_DIM - 1) {
    partialSums[blockIdx.x] = inBuffer_s[threadIdx.x];
}

output[i] = inBuffer_s[threadIdx.x];
```


Slide credit: Izzat El Hajj

Work Efficiency

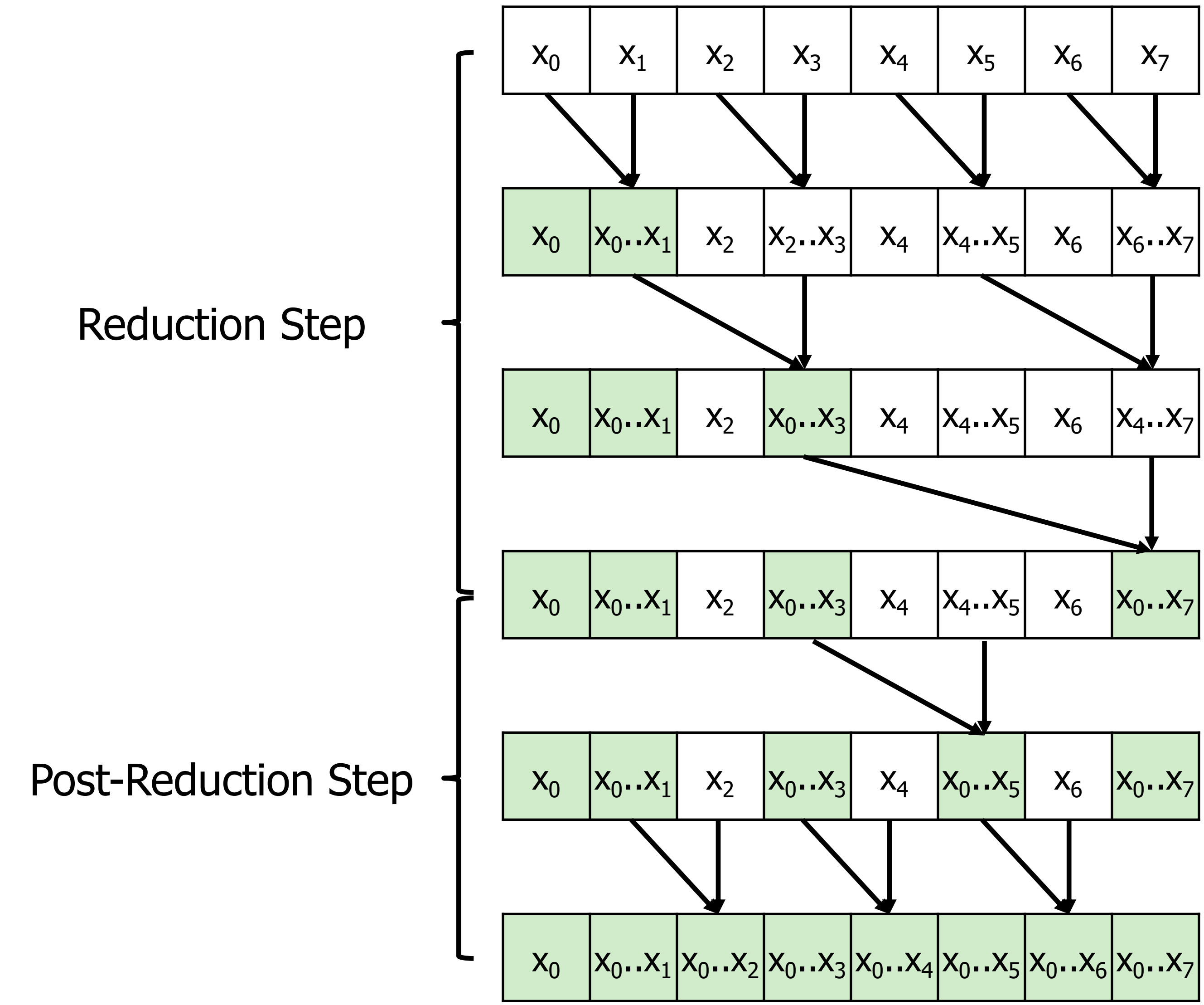
- A parallel algorithm is work-efficient if it performs the same amount of work as the corresponding sequential algorithm
- Scan work efficiency
  - Sequential scan performs N additions
  - Kogge-Stone parallel scan performs:
    - log(N) steps, N - 2^{step} operations per step
    - Total: (N-1) + (N-2) + (N-4) + … + (N-N/2)

      = N*log(N) - (N-1) = O(N*log(N)) operations
    - Algorithm is not work efficient
- If resources are limited, parallel algorithm will be slow because of low work efficiency



Slide credit: Izzat El Hajj

Brent-Kung Parallel (Inclusive) Scan

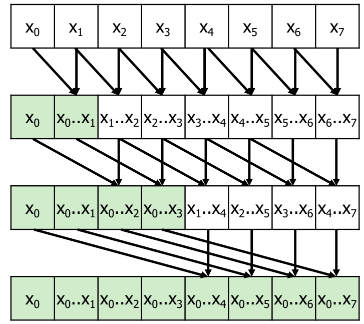


One thread for every two elements

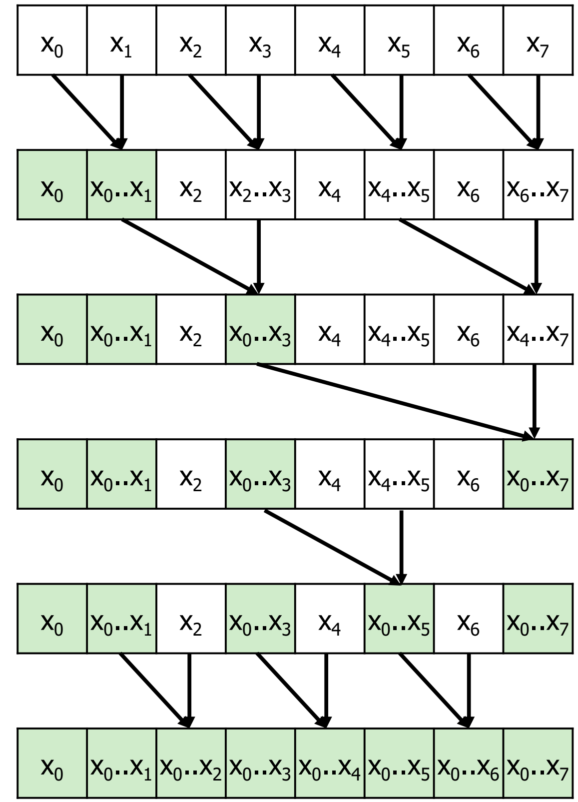
Slide credit: Izzat El Hajj

Work Efficiency

- Recall: Kogge-Stone
  - log(N) steps
  - O(N*log(N)) operations



- Brent-Kung



  - Reduction step:
    - log(N) steps
    - 1 + 2 + 4 + … + N/2 = N-1 operations
  - Post-Reduction step:
    - log(N)-1 steps
    - (2-1) + (4-1) + … + (N/2-1) = (N-2) - (log(N)-1)
  - Total:
    - 2*log(N)-1 steps
    - (N-1) + (N-2) - (log(N)-1) = 2*N – log(N) – 2 = O(N) operations
- Brent-Kung takes more steps but is more work-efficient

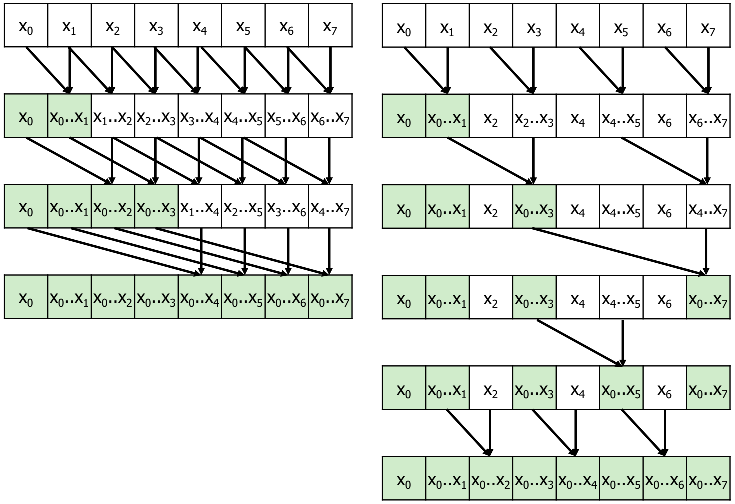
Slide credit: Izzat El Hajj

Work Efficiency (the Reality)

- While Brent-Kung has higher theoretical work-efficiency than Kogge-Stone, in practice, its actual resource consumption on GPUs after accounting for inactive threads is O(N*log(N))

- Performance of Brent-Kung on GPUs is similar or may even be worse than Kogge-Stone

- Still is an interesting case to study



Slide credit: Izzat El Hajj

Recall: Warp Shuffle Functions

- Built-in warp shuffle functions enable threads to share data with other threads in the same warp
  - Faster than using shared memory and `__syncthreads()` to share across threads in the same block
- Variants:
  - `__shfl_sync(mask, var, srcLane)`
    - Direct copy from indexed lane
  - `__shfl_up_sync(mask, var, delta)`
    - Copy from a lane with lower ID relative to caller
  - `__shfl_down_sync(mask, var, delta)`
    - Copy from a lane with higher ID relative to caller
  - `__shfl_xor_sync(mask, var, laneMask)`
    - Copy from a lane based on bitwise XOR of own lane ID

Slide credit: Izzat El Hajj

Recall: Per-Block (Inclusive) Scan

- Inside a thread block, we can also apply a hierarchical approach
  - Warps
  - Threads

- Let’s start with the basic algorithms
  - Kogge-Stone
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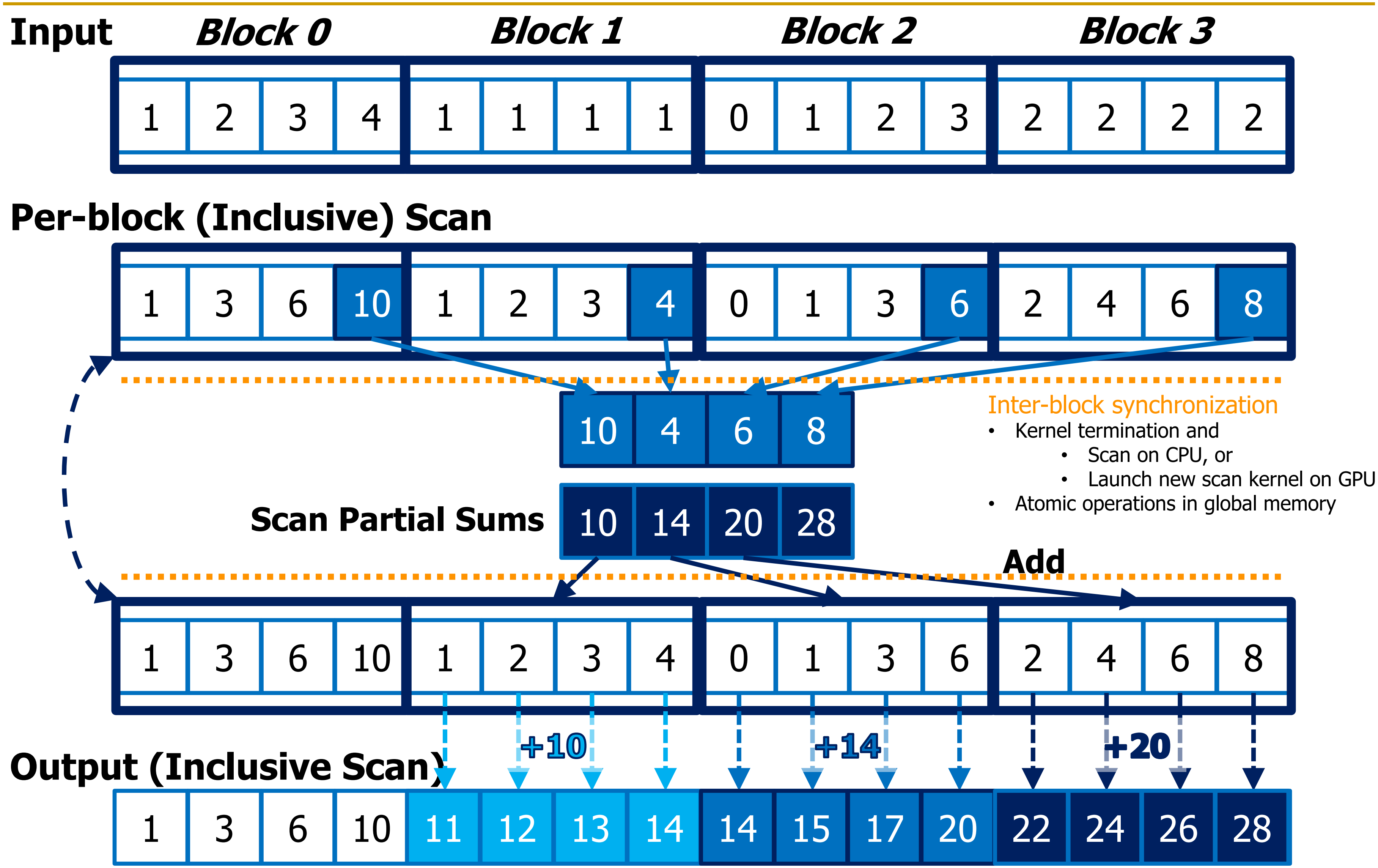
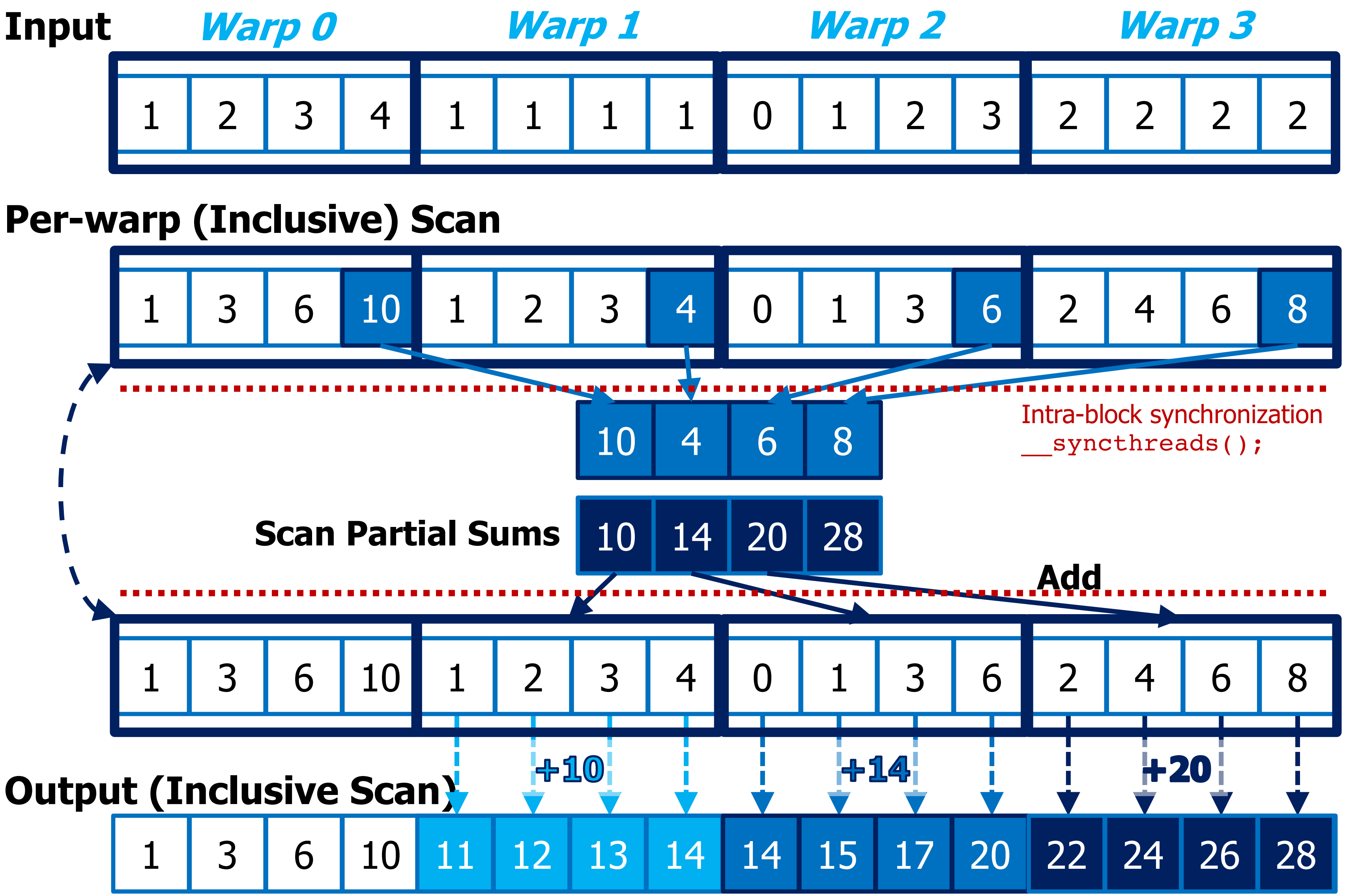
Recall: Hierarchical (Inclusive) Scan

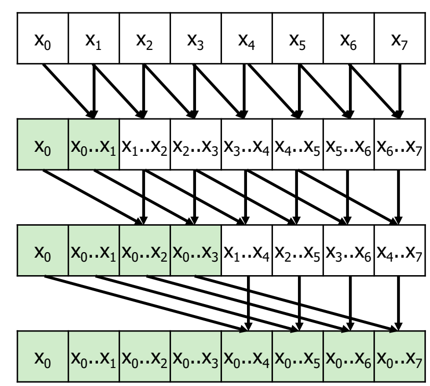


Per-Block Hierarchical (Inclusive) Scan



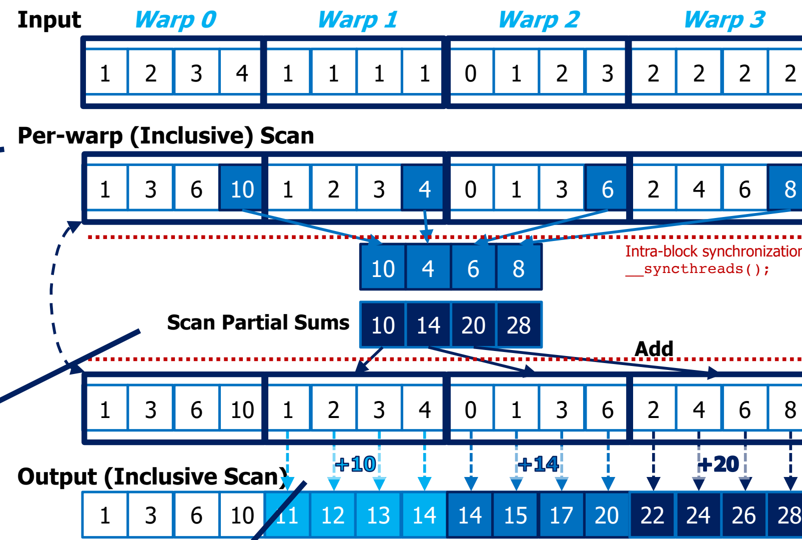
Warp Scan

```
#define WARP_SIZE 32

// Warp ID and lane ID
__device__ inline int lane_id(void) { return threadIdx.x % WARP_SIZE; }
__device__ inline int warp_id(void) { return threadIdx.x / WARP_SIZE; }


// Warp scan
__device__ int warp_scan(int val){
  int x = val;

  #pragma unroll
  for(int offset = 1; offset < WARP_SIZE; offset <<= 1){

    int y = __shfl_up_sync(0xffffffff, x, offset);

    if(lane_id() >= offset)
      x += y;
  }

  return x - val;
}
```

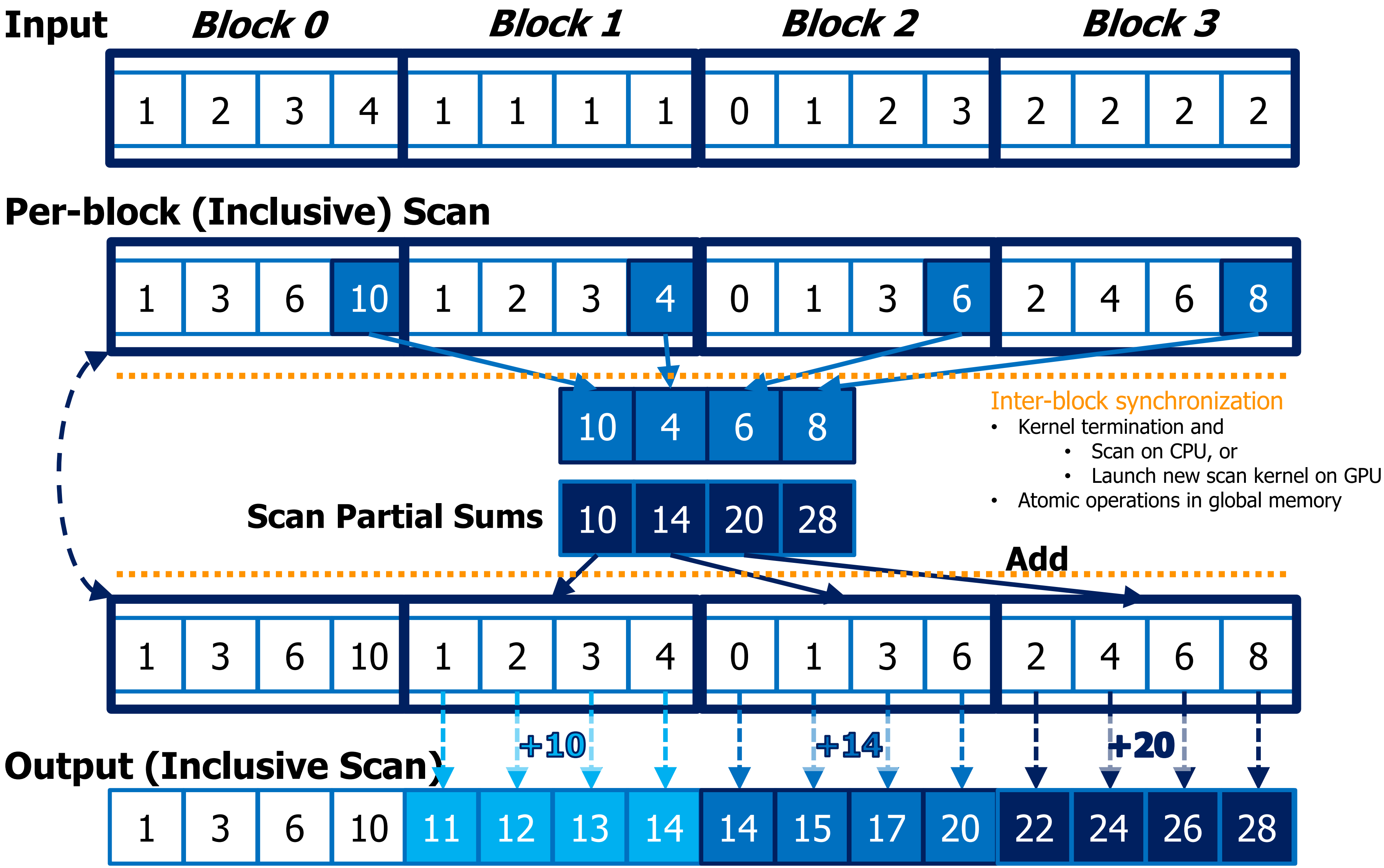


Per-Block Hierarchical Scan

```
__device__ int block_scan(int* count, int x){

  __shared__ int sdata[L_DIM];

  // A. Exclusive scan within each warp
  int warpPrefix = warp_scan(x);

  // B. Store in shared memory
  if(lane_id() == WARP_SIZE - 1)
    sdata[warp_id()] = warpPrefix + x;

  __syncthreads();

  // C. One warp scans in shared memory
  if(threadIdx.x < WARP_SIZE)
    sdata[threadIdx.x] = warp_scan(sdata[threadIdx.x]);

  __syncthreads();

  // D. Each thread calculates it final value
  int thread_out_element = warpPrefix + sdata[warp_id()];
  int output = thread_out_element + *count;

  __syncthreads();

  if(threadIdx.x == blockDim.x - 1)
    *count += (thread_out_element + x);

  return output;
}
```

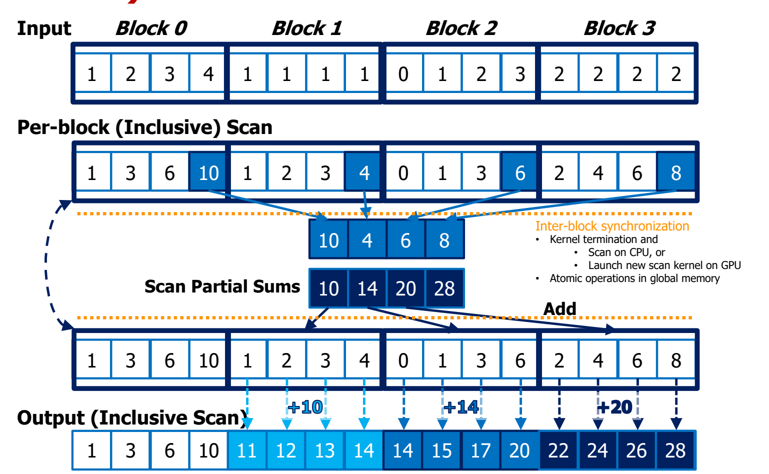


Scan-Scan-Add (SSA)

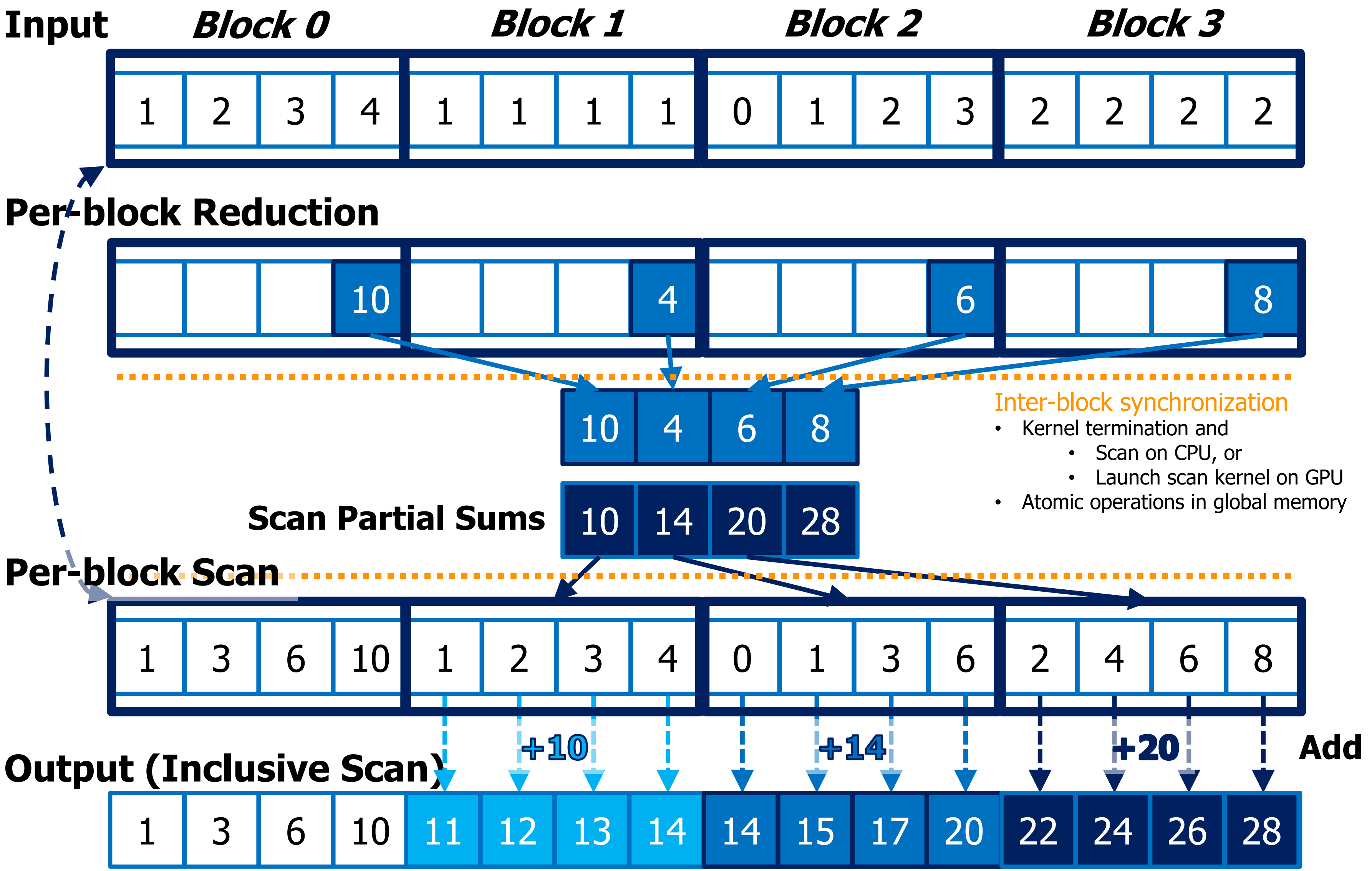


SSA: Global Memory Accesses

- Scan
  - First kernel reads input array (N elements) and writes array with per-block prefix sums (N elements)
- Scan
  - Second kernel reads and writes N / BLOCK_SIZE elements
- Add
  - Third kernel reads array with per-block prefix sums (N elements) and writes output (N elements)
- 4N elements are read/written

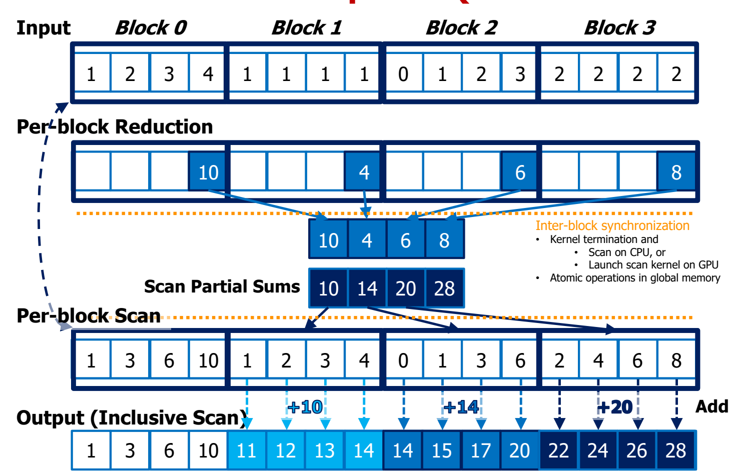


Reduce-Scan-Scan (RSS)



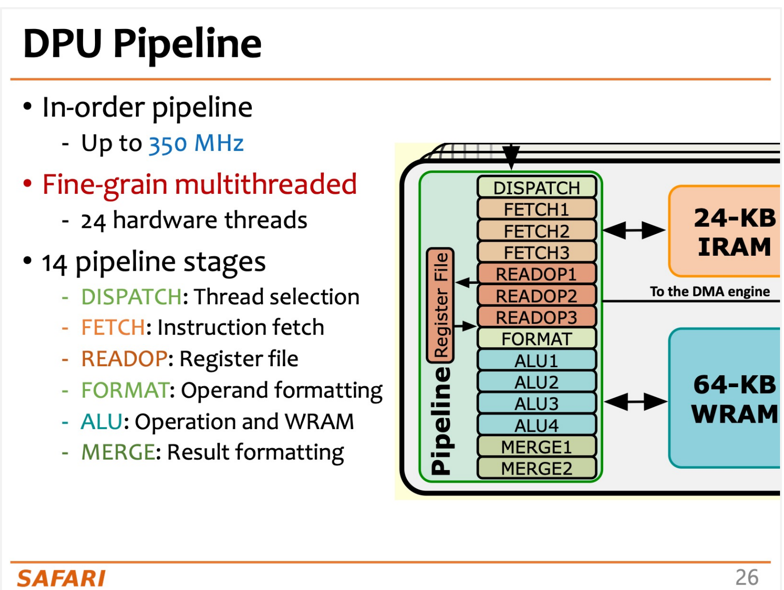
RSS: Global Memory Accesses

- Reduce
  - First kernel reads input array (N elements) and writes per-block reduction (N / BLOCK_SIZE elements)
- Scan
  - Second kernel reads and writes N / BLOCK_SIZE elements
- Scan
  - Third kernel reads input array (N elements) and scan partial sums (N / BLOCK_SIZE elements), and writes output (N elements)
- 3N elements are read/written



SSA vs. RSS in a FGMT Architecture

- UPMEM Processing-in-Memory cores are fine-grained multithreaded
- Threads (called *tasklets*) can use handshakes to communicate pairs of tasklets and barriers to synchronize all tasklets



Processing-in-Memory Course
Meeting 2: Real-world PIM architectures (Fall 2021)
https://youtu.be/D8Hjy2iU9l4

Benchmarking a New Paradigm: An Experimental Analysis of a Real Processing-in-Memory Architecture

Juan Gómez-Luna[1] Izzat El Hajj[2] Ivan Fernandez[1,3] Christina Giannoula[1,4] Geraldo F. Oliveira[1] Onur Mutlu[1]

[1]ETH Zürich [2]American University of Beirut [3]University of Malaga [4]National Technical University of Athens

https://arxiv.org/pdf/2105.03814.pdf

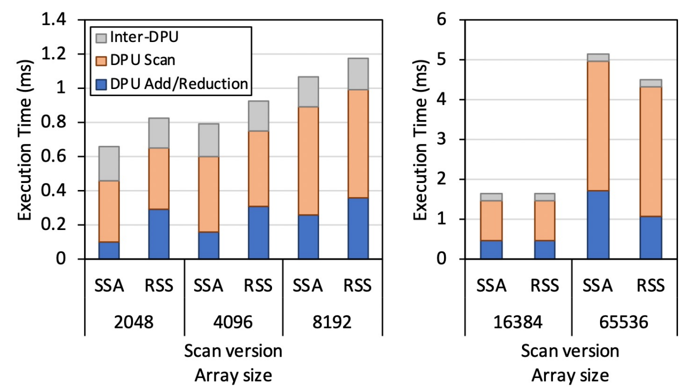
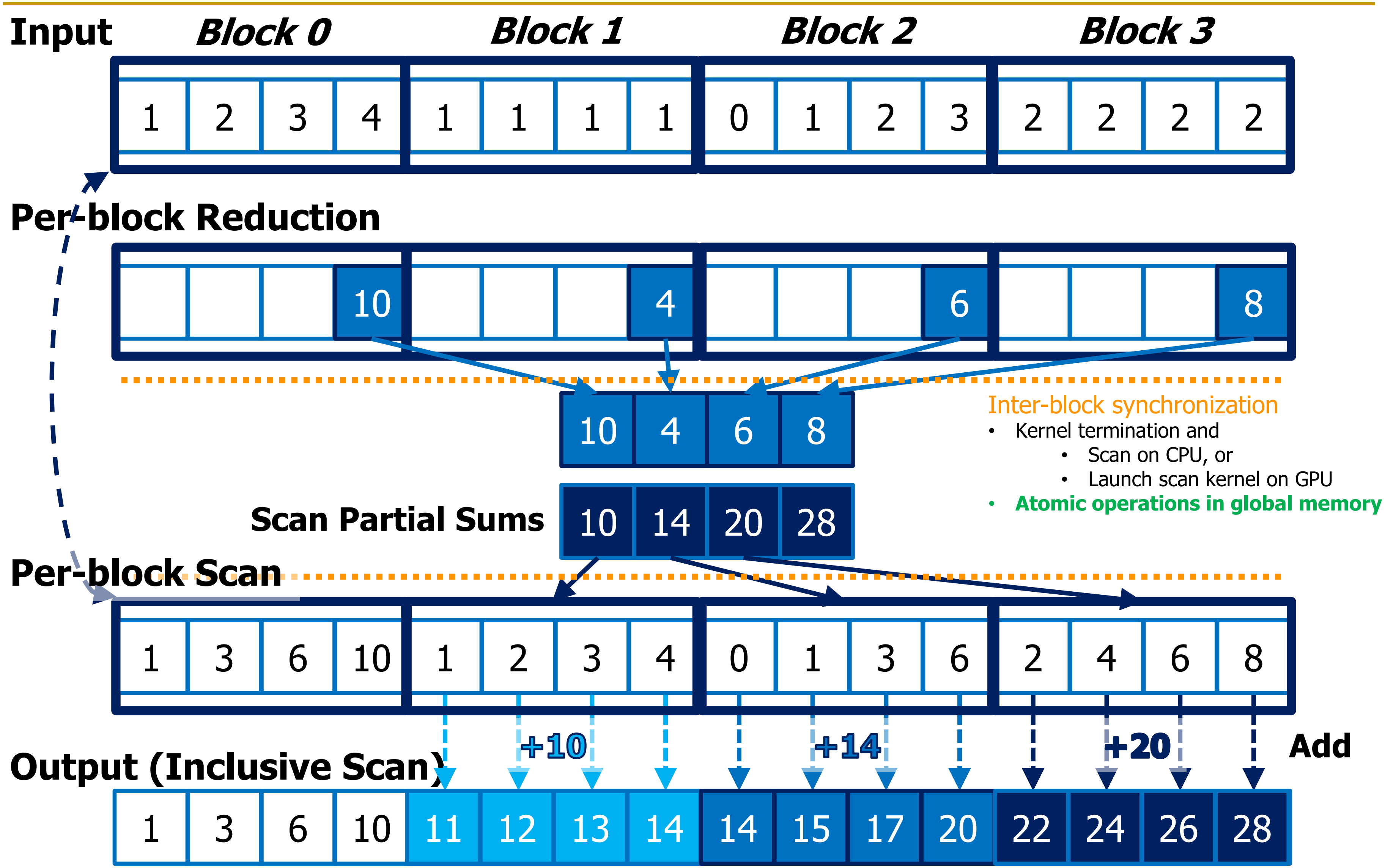


Figure 24: Two versions of scan (SCAN-SSA, SCAN-RSS) on 1 DPU.

Reduce-Scan-Scan (RSS)



Adjacent Block Synchronization (I)

```
__shared__ float previous_sum;

if (threadIdx.x == 0){

  // Wait for previous flag
  while (atomicAdd(&flags[bid], 0) == 0){;}

  // Read previous partial sum
  previous_sum  =  scan_value[bid];

  // Propagate partial sum
  scan_value[bid + 1] = previous_sum + local_sum;

  // Memory fence
  __threadfence();

  // Set flag
  atomicAdd(&flags[bid + 1], 1);
}
__syncthreads();
```

`flags` and `scan_value` reside in global memory

Adjacent Block Synchronization (II)

```
__shared__ float previous_sum;

if (threadIdx.x == 0){

  // Wait for previous flag
  while (atomicAdd(&flags[bid], 0) == 0){;}

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  // Propagate partial sum
  scan_value[bid + 1] = previous_sum + local_sum;

  // Memory fence
  __threadfence();

  // Set flag
  atomicAdd(&flags[bid + 1], 1);
}
__syncthreads();
```

Thread blocks may not scheduled linearly (in accordance with their block ID)

In the beginning of the kernel, we obtain a dynamic block ID:

```
__shared__ int sbid;

if (threadIdx.x == 0)
  sbid = atomicAdd(DCounter, 1);

__syncthreads();

const int bid = sbid;
```

Adjacent block synchronization reduces global memory accesses from 3N (or 4N) to 2N elements

`DCounter` resides in global memory

Recall: Scan Applications

- Scan is a key parallel primitive that can
  - convert recurrences from sequential

```
for(int i=1; i<n; i++)
    out[i] = out[i-1] + f(i);
```

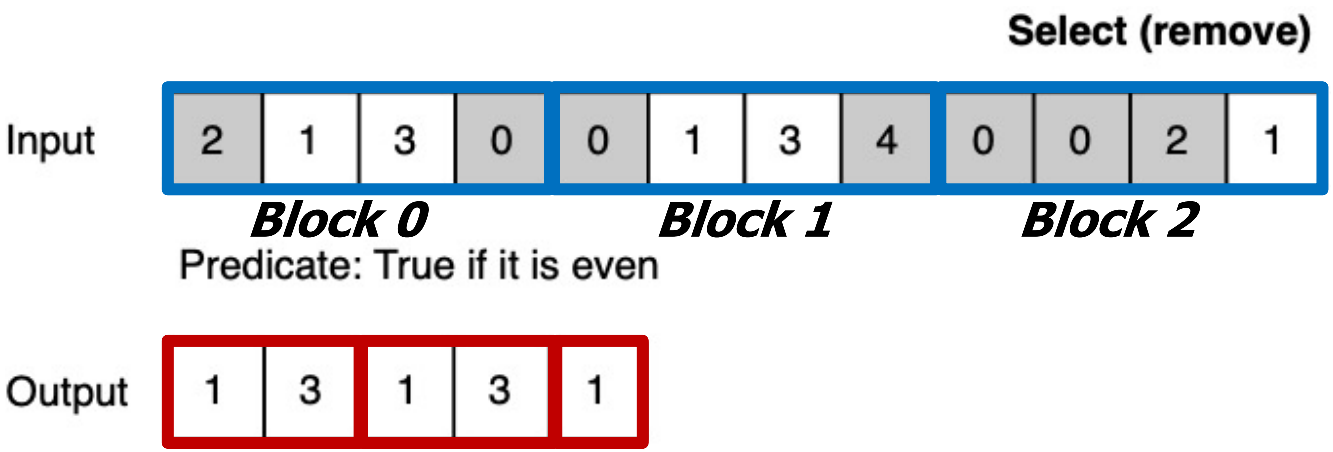
  - into parallel

```
forall(i) {temp[i] = f(i)};
scan(out, temp);
```

- Scan is a basic building block of many parallel algorithms
  - E.g., stream compaction, partition, select, unique, radix sort, quicksort, string comparison, lexical analysis, polynomial evaluation, solving recurrences, tree operations, histograms, etc.

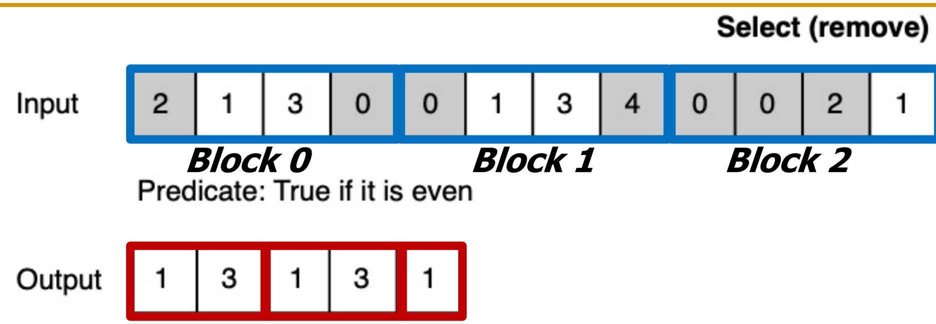
In-Place Data Sliding Algorithms (I)

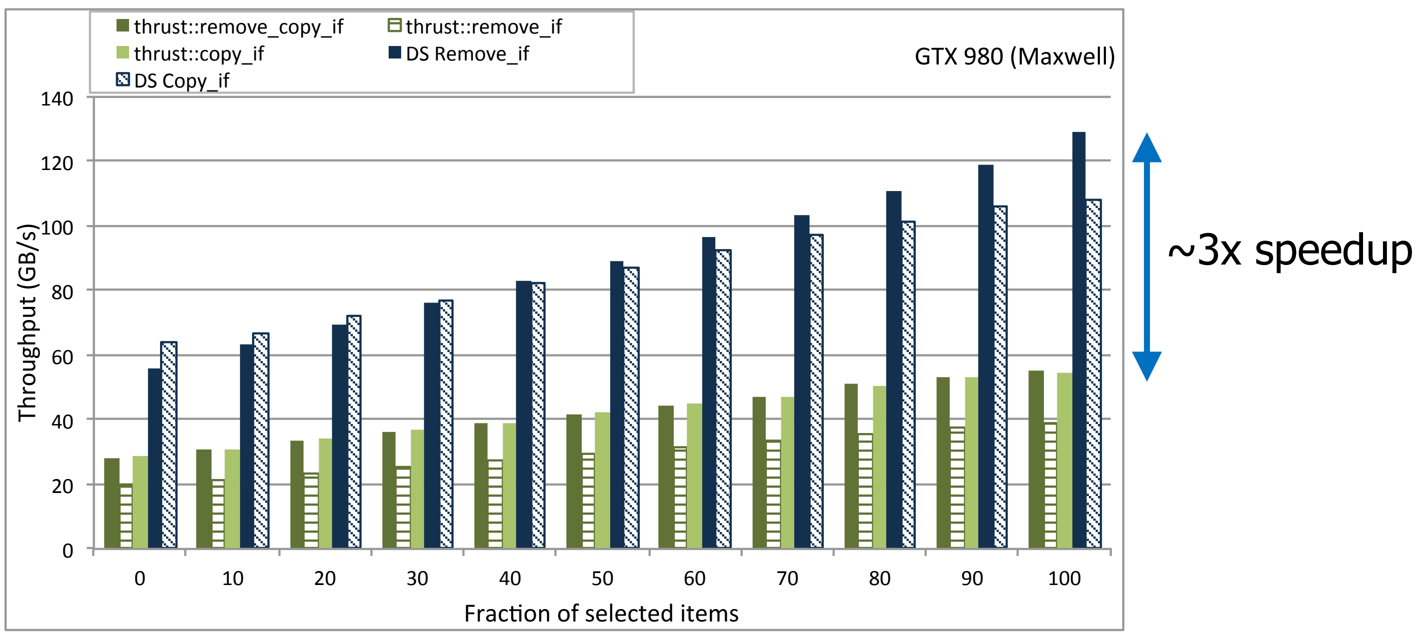
- Algorithms that move data in memory in one direction
  - Padding / Unpadding
  - Stream compaction
  - SELECT
  - UNIQUE
  - Partitioning
  - Etc.



In-Place Data Sliding Algorithms (II)

- SELECT



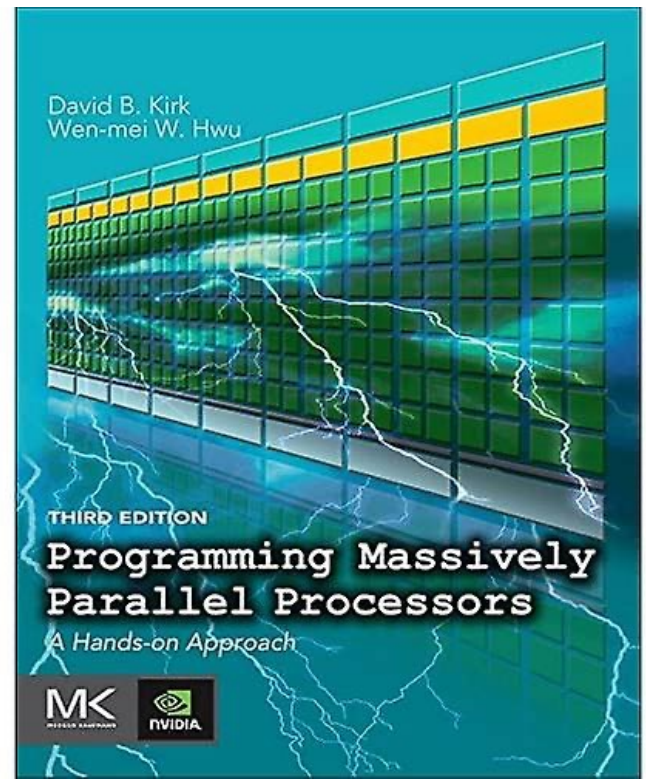


Recommended Readings

- Hwu and Kirk, "Programming Massively Parallel Processors," Third Edition, 2017
  - Chapter 8 - Parallel patterns:

  prefix sum: An introduction to

  work efficiency in parallel algorithms



P&S Heterogeneous Systems

Parallel Patterns: Prefix Sum (Scan)

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